Corrections to
LECTURES ON SOME ASPECTS OF THE BOOTSTRAP

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This note contains corrections to my 1996 St-Flour Lecture Notes on the bootstrap (Giné, 1997), and also corrects a few misprints. Page numbers refer to the Springer Lecture Notes. The page numbers in the PDF version available in this page are smaller by 36.

1) The proof of Lemma 2.5 in Chapter 1, pages 56–57, contains an incorrect statement, that of lines 3 to 6 on page 57. However, it is certainly true, by exchangeability, that, for each $\omega \in \Omega$, the conditional laws given ω of the two random variables $\bar{X}_n^*(\omega) - \bar{X}_n(\omega)$ and $\sum_{i=1}^n \big(w_n(i) - \frac{1}{n}\big) X_{(n,i)}(\omega)$ coincide and this fact is all that is needed to conclude that, to prove the lemma, it suffices to prove that for all ω_1 and ω_2 in Ω_1,

$$n\mathbf{E}_w \Big(\sum_{i=1}^n \big(w_n(i) - \frac{1}{n}\big) \big(X_{(n,i)}(\omega_2) - X_{(n,i)}(\omega_1) \Big)^2 \to 0.$$

The proof of this is carried out on page 57, after line 10.

2) Theorem 1.20 in Chapter 2, page 102, is wrong as stated, and I thank R. Dudley for noticing this. In order to make the theorem correct, the pseudometric must be totally bounded (as opposed to separable) and an extra hypothesis is needed, marked with a $(*)$ in the following statement that should replace that theorem:

1.20. THEOREM *corrected.* *Let (T, d) be a totally bounded pseudo–metric space and, for each $n \in \mathbf{N}$, let $X_{n,1}(t), \dots, X_{n,n}(t)$, $t \in T$, be i.i.d. processes with bounded sample paths, defined on different factors of a product probability space. Assume*
0) for all $\lambda > 0$,

$$\lim_{n\to\infty} \sup_{t\in T} \Pr\big\{ |X_{n,1}(t)| > \lambda\sqrt{n} \big\} = 0 \qquad (*)$$

i) for all t in a d–dense subset D of T and for all $\lambda > 0$,

$$\lim_{n\to\infty} n\Pr^*\big\{ |X_{n,1}(t)| > \lambda\sqrt{n} \big\} = 0, \qquad (1.32)$$

and ii)

$$\lim_{\delta\to 0} \limsup_{n\to\infty} \Pr^* \Big\{ \frac{1}{\sqrt{n}} \sup_{d(s,t)\le\delta} \Big| \sum_{i=1}^n \big(X_{n,i}(t) - \mathbf{E}X_{n,i}(t) - X_{n,i}(s) + \mathbf{E}X_{n,i}(s) \big) \Big| > \varepsilon \Big\} = 0 \qquad (1.33)$$

for all $\varepsilon > 0$.
Then,

$$\lim_{n\to\infty} n\Pr^*\Big\{\big\|X_{n,1}\big\|_T > \lambda\sqrt{n}\Big\} = 0 \tag{1.34}$$

for all $\lambda > 0$. If hypotheses (*), 1) and 2) hold only along a subsequence n_k, then the same is true for the conclusion.

The proof in Giné (1997) actually uses total boundedness of (T, d), and it is correct for the symmetrized processes, that is, the proof of (1.39) there is correct. The error occurs when we desymmetrize, below (1.39) on page 103. One must proceed as follows: the symmetrization inequality in Lemma 2.5 of Giné and Zinn (1984) implies

$$n\Pr^*\Big\{\big\|Y_{n,1}\big\|_T > \lambda\sqrt{n}\Big\} \geq n\Pr^*\Big\{\big\|X_{n,1}\big\|_T > 2\lambda\sqrt{n}\Big\}\Big[1-\sup_{t\in T}\Pr\big\{|X_{n,1}(t)| > \lambda\sqrt{n}\big\}\Big].$$

Then, (1.39) and condition (*) give (1.34), proving the theorem.

Theorem 1.20 is used on page 110, in the proof of the converse part of the bootstarp theorem 2.2, in Chapter 2. The new version still applies since, a) $(\mathcal{F}, \rho_P)$ is totally bounded (and not just separable) by Remark 1.2 on page 85, and b) condition (*) in this case, that is,

$$\lim_{n\to\infty} \Pr^b\big\{|f(X^b_{n,1}(\omega))| > \lambda\sqrt{n}\big\} = 0 \quad a.s.,$$

also holds true because, for each n, these probabilities are dominated by the random variable $n^{-1}\sum_{i=1}^n I_{F(X_i)>\lambda\sqrt{n}}$, which tends to zero a.s. by the law of large numbers.

Here are a few misprints and completions of arguments:

Pages 45, 46: the numbering 1.7 appears twice, but no confusion arises because one of them is always referred as 'Proposition 1.7'.

Page 54, line 2 from bottom: replace '$BL(\mathbf{R})$ is separable' by '$BL([-n, n])$ is totally bounded for the sup norm for all n, where BL_1 of a set is actually defined later, in the middle of page 77'.

Page 55, line 12: $w_{n'}(j, \omega')$ should be replaced by $w_{n'}(i, \omega')$.

Page 69, equation (4.8), 'f' should be replaced by 'f^2', twice.

Page 92, lines 11 and 13: the '+' signs must be replaced by '–' signs (three times!).

Page 96, line 7: 'Pr' should be replaced by 'Pr*'.

Page 99: The proof of Theorem 1.17, b) implies c), is too short. It is based on Theorem 1.16 and Hoffmann–Jørgensen's inequality, as stated, but it requires the fact that $\sup_{t>0} t^2 \Pr^*\{F_c(X) > t\} < \infty$ implies that

$$\sup_n \mathbf{E}^* \max_{i\leq n}\left(\frac{F_c(X_i)}{\sqrt{n}}\right)^{2-\delta} < \infty$$

for all $0 < \delta < 2$, and this can be seen by integration by parts.

Page 105, line 3 from bottom: 'inequalities (1.16) and (1.17)' should be replaced by 'inequalities (1.11) and (1.13)'.

Page 106, lines 5 and 6 from bottom: '$\mathbf{E}\left\| \sum_{\mathbf{i=1}}^{\mathbf{n}} \mathbf{X_i} \right\|$' should be twice replaced by $\mathbf{E}^* \left\| \sum_{i=1}^{n} X_i \right\|_T$.

Page 121, line 11: Replace 'If S is a random permutation then' by 'If S is a random permutation independent of R, then'.

Page 121, line 5 from bottom: replace (2.7) by (2.10).

Page 138, line 2 from bottom: a parenthesis must be replaced by a norm sign.

References

Giné, E. (1997). Lectures on some aspects of the bootstrap. In *Lectures on Probability and Statistics: Ecole d'Eté de Probabilitiés de Saint–Flour, XXVI–1996*, Ed.: P. Bernard, *Lecture Notes in Math.* **1665**, pp. 37-151, Springer Verlag, Berlin.

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