Problems

Problems contain propositions from Book I of Euclid’s Elements. In each instance, prove the indicated result.

1. Proposition 6. If two angles of a triangle are congruent with one another, then the sides opposite these angles will also be congruent. [Hint: Let $ABC$ be a triangle in which $\angle CAB = \angle CBA$. If $AC \neq BC$, say, $AC > BC$, then choose a point $D$ on $AC$ such that $AD = BC$.]

2. Proposition 15. If two lines cut one another, then they make vertical angles that are equal. [Hint: Appeal to Proposition 13, which says that if a ray is drawn from a point on a line, then the sum of the pair of supplementary angles formed is equal to two right angles.]

3. Proposition 17. In a triangle, the sum of any two angles is less than two right angles. [Hint: In $\triangle ABC$, extend segment $BC$ to a point $D$ and use the exterior angle theorem.]

4. Proposition 18. If one side of a triangle is greater than a second side, then the angle opposite the first is greater than the angle opposite the second. [Hint: In $\triangle ABC$, for $AC > AB$, choose a point $D$ on $AC$ such that $AD = AB$; use the fact that $\angle ADB$ is an exterior angle of $\triangle BCD$.]

5. Proposition 26. Two triangles are congruent if they have one side and two adjacent angles of one congruent with a side and two adjacent angles of the other. [Hint: Let $\triangle ABC$ and $\triangle DEF$ be such that $\angle B = \angle E$, $\angle C = \angle F$, and $BC = EF$. If $AB \neq DE$, say $AB > DE$, choose a point $G$ on $AB$ for which $BG = ED$.]
Solutions to Euclid Problems

1. Triangles $DAB$ and $CBA$ are congruent by the side-angle-side theorem; hence, $\angle DBA = \angle CAB = \angle CBA$, which contradicts Common Notion 5.

2. $\alpha + \beta = 180^\circ = \beta + \gamma$ implies that $\alpha = \gamma$.

3. Because $\angle ABC < \angle ACD$ by the exterior angle theorem, it follows that $\angle ABC + \angle ACD < \angle ACD + \angle ACB = 180^\circ$.

4. Triangle $ABD$ is isosceles, hence $\angle ABD = \angle ADB$. Applying the exterior angle theorem, $\angle ABC > \angle ABD = \angle ADB > \angle ACB$.

5. Triangles $GBC$ and $DEF$ are congruent by the side-angle-side theorem; hence, $\angle C = \angle F = \angle BCG$, which contradicts Common Notion 5.