Math 2210Q-004 **Quiz Questions** **Spring 2009**

For Review Purposes

1. Suppose A and B are 4×4 matrices where the last column of AB is the zero vector, but where no column of B is the zero vector. What can you say about the columns of A and how does that impact on whether or not A is invertible? Explain your reasoning.

   Remark: It might be easier to explain if you write $B = [\vec{b}_1\ \vec{b}_2\ \vec{b}_3\ \vec{b}_4]$.

2. Express $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ as a product of elementary matrices.

3. Without finding the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 4 & 7 \end{bmatrix}$, find the 2^{nd} column of A^{-1}. Show all your work.

4. If the columns of a matrix P are linearly independent, what can you say about the solution(s) of the nonhomogeneous system $P\vec{x} = \vec{b}$? Explain why?

5. If A is an invertible matrix, must A^2 also be invertible? Explain why or why not.

6. If A is a square matrix with linearly independent columns, then the rows of A are also linearly independent. Explain why this is so.

7. Using your knowledge of invertible matrices, explain without row reduction computations why $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ is or is not invertible.

8. Suppose that $A^{-1} = \begin{bmatrix} -3 & 0 & 2 \\ -6 & 1 & 3 \\ 2 & 0 & -1 \end{bmatrix}$. Solve the system $A\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

9. Determine whether each of the following matrices is invertible. No arithmetic is needed; explain your answers.

   (a) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 4 & 5 & 6 \end{bmatrix}$