For problems 1-27, evaluate the following indefinite, definite and improper integrals using geometry, the fundamental theorem of calculus possibly along with u-substitution or tables of integrals or all three. If the improper integral does not converge, show why.

1. \( \int \frac{25}{3x + 1} \, dx \)
2. \( \int x(6x^2 + 2)^{10} \, dx \)
3. \( \int \frac{e^x}{\sqrt{x^2 + 1}} \, dx \)
4. \( \int (x^2 + 1)(4x^3 + 12x)^5 \, dx \)
5. \( \int \frac{\sqrt{\ln x}}{x} \, dx \)
6. \( \int 7x^2 e^{x^3+2} \, dx \)
7. \( \int (5x^2 + 2x + 8) \, dx \)
8. \( \int \frac{s + 1}{s^2 + 2s} \, ds \)
9. \( \int \frac{e^x + e^{-x}}{e^x - e^{-x}} \, dx \)
10. \( \int \frac{1}{t \ln \sqrt{t}} \, dt \)
11. \( \int_0^2 (x^4 + \sqrt{x}) \, dx \)
12. \( \int_0^1 xe^{x^2+1} \, dx \)
13. \( \int_{-2}^{3} (1 - |x|) \, dx \)
14. \( \int_0^3 \sqrt{9 - x^2} \, dx \)
15. \( \int_1^{\infty} \frac{1}{x^2} \, dx \)
16. \( \int_1^{\infty} \frac{1}{x} \, dx \)
17. \( \int_0^{\infty} xe^{-x^2} \, dx \)
18. \( \int_{-\infty}^{0} xe^{-x^2} \, dx \)
19. \( \int_{-\infty}^{\infty} xe^{-x^2} \, dx \)
20. \( \int_0^4 \frac{1}{\sqrt{x^2 + 1}} \, dx \)
21. \( \int_0^1 \frac{1}{4 - x^2} \, dx \)
22. \( \int x^2 e^{-2x} \, dx \)
23. \( \int x \left( \ln(x^2 + 1) \right)^2 \, dx \)
24. \( \int_0^{\infty} x^2 e^{-x^3} \, dx \)
25. \( \int_{-\infty}^{0} x^2 e^{-x^3} \, dx \)
26. \( \int_{-\infty}^{\infty} x^2 e^{-x^3} \, dx \)

27. Consider the function \( f(x) = \begin{cases} 
2x^2 & -2 \leq x < 2 \\
x^3 & 2 < x \leq 5
\end{cases} \)

(a) Find \( \int_{0}^{4} f(x) \, dx \)
(b) What is the average value of \( f(x) \) over the interval \([0, 4]\).

28. (a) Use the left hand sum with \( n = 6 \) to approximate \( \int_{0}^{3} \sqrt{x^2 + 1} \, dx \)
(b) Is your answer above an over-estimate or an under-estimate for the true value of the definite integral \( \int_{0}^{3} \sqrt{x^2 + 1} \, dx \)
29. Calculate $\int_{1}^{4} 13 - 2x \, dx$ first by geometric means and then verify your answer by using the fundamental theorem of calculus.

30. Find the cost function for a hairbrush manufacturer if the marginal cost, in dollars, is given by $x^3(100 + x^4)$, where $x$ is the number of cases of brushes produced and the fixed costs are $1,000.$

31. Suppose a positive-valued continuous function $y = f(x)$ is decreasing over the interval $[0, 4]$. Using the following table of values for $f$,

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>50</td>
<td>40</td>
<td>25</td>
<td>10</td>
<td>5</td>
</tr>
</tbody>
</table>

Give an upper and a lower estimate for the area under the graph of $y = f(x)$ over the interval $[0, 4]$. Explain what you’re doing.

32. Use the fundamental theorem of calculus to find the derivative of the following functions:
   (a) $G(x) = \int_{1}^{x} \frac{1}{t} \, dt$
   (b) $H(x) = \int_{-1}^{x} \sqrt{1 + t^2} \, dt$
   (c) $G(x^2 + 1)$ where $G(x) = \int_{1}^{x} \frac{1}{t} \, dt$ as above.

33. After the introduction of a new camera, the rate of sales in thousands per month is given by $N'(t) = 10(1 + 2t)^{-\frac{5}{2}}$, where $t$ is in months. How many cameras were sold in the first 4 months?

34. Suppose Galileo drops a pebble from the Leaning Tower of Pisa, which stands at a height of 58.4 meters. Given that the pebble falls vertically with acceleration $a(t) = -9.8 \text{ m/sec}^2$, how long does it take for the pebble to reach the ground? (Hint: $a(t) = \frac{d}{dt}v(t)$ where $v(t)$ is the velocity of the pebble at time $t$ and $v(t) = \frac{d}{dt}h(t)$ where $h(t)$ is the height of the pebble at time $t$.)

35. An object travels with a velocity $v(t) = \frac{1}{t}$ where $1 \leq t \leq 3$. Give an upper and a lower estimate for the distance traveled by the object on the interval $[1, 3]$ by dividing the interval into $n = 6$ intervals of equal length. Compute the exact distance the object traveled on $[1, 3]$.

36. Find the area enclosed by the curves $y = x^2$ and $y = 2x$.

37. Find the area enclosed by the curves $y = |x|$, $y = 3$, $x = -1$ and $x = 6$.

38. Find the area enclosed by the curves $y = e^x$, $y = e^{-x}$ and $y = 2$. 

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39. Let \( f(t) = (0.02t)e^{(0.01t^2 + 0.05t)} \) be the rate of change of the total flow of income where \( t \geq 0 \) is in years. Given an interest rate \( r = 5\% \) compounded continuously, what is the present value of the income flow at time \( T \)? Suppose that the entire income flow is compounded continuously at interest rate \( r = 5\% \), what is the amount of money after \( T \) years?

40. Consider an investment from an income flow of \( 100e^{0.25t} \), where \( t \) is measured in years that will last 20 years.

   (a) How much will this income flow amount to at the end of the 20 years?
   
   (b) How much is the investment worth today assuming a constant interest of 6\% continuously compounded interest rate for the duration of the investment?

41. An investment with a continuous income flow of \( 15,000e^{0.04t} \) in dollars per year forever is to be sold. Assume that a constant rate of interest of 7\% compounded continuously will likewise continue forever. How much should the investment be sold for today?