University of Connecticut
Department of Mathematics

Math 210Q	Exam III: Integrals	V0 - FS - 2006
Name:		Section: 002

Read This First !

- Use of a pocket calculator or graphing calculator is only allowed if the calculator cannot perform alpha numeric calculations. Storing of any formulas in memory is forbidden.
- If part of the solution is written outside the space provided for this solution clearly indicate this.
- In the first question you can split up the iterated integral as a product of integrals. For the one over x you will have to use integration by parts where u equals x and dv is the rest.
- The last page contains a few formulas. Look at it before you start!
- There is 10% extra credit in here. Good Luck! ☺

Grading - For Administrative Use Only

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1. **Double Integrals I** *(10 points)*

Calculate $\int_0^1 \int_1^2 \frac{xe^x}{y}\, dy\, dx$.

2. **Double Integrals II** *(15 points)*

Calculate $\displaystyle\iint\limits_D \frac{4y}{x^3+2}\,dA$ where $D = \{(x,y)\ :\ 1 \le x \le 2, 0 \le y \le 2x\}$.

3. **Double Integrals III** *(15 points)*

Calculate $\displaystyle\iint_D \arctan\left(\frac{y}{x}\right)\ dA$ where $D = \{(x, y)\ :\ 1 \leq x^2 + y^2 \leq 4, 0 \leq y \leq x\}$.

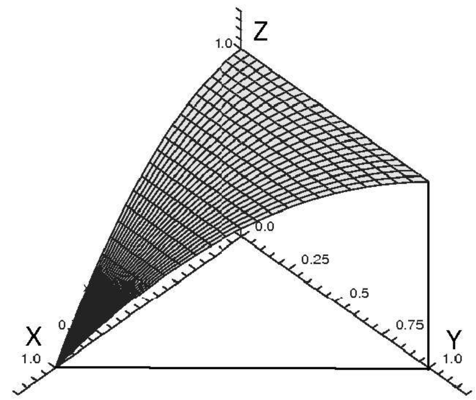
4. **Surfaces** *(15 points)*

Find the area of the surface $z = xy$ inside the cylinder $x^2 + y^2 = 4$.

5. **Triple Integrals I** *(20 points)*

Evaluate the triple integral $\iiint_E y\,dV$ where E is a solid described by

$E = \{(x, y, z) \ : \ x \geq 0, y \geq 0, z \geq 0, y \leq 1 - x \text{ and } z \leq 1 - x^2\}.$



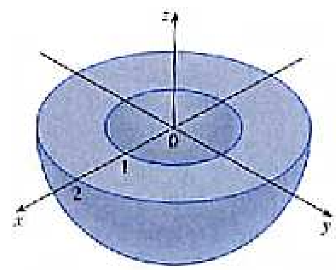
6. **Triple Integrals II** *(20 points)*

Evaluate the triple integral

$$\iiint_E y^2\ (x^2+y^2+z^2)\ dV$$

where E is the solid described as follows:

- All points are outside a sphere with radius 1.
- All points are inside a sphere with radius 2.
- All points are below the XY plane.



Both spherical and cylindrical coordinates will work, but one is noticeably easier than the other. Don't try to do this in Cartesian coordinates. You would suffer. A lot. Really.

7. **Setting up an Integral** *(7 points)*

Set up, but do not calculate the integral of $f(x, y, z) = 7x^2$ over
$E = \{(x, y, z)\ :\ 9 \leq x^2 + y^2 \leq 16,\ 5 \leq z \leq 6,\ y \leq 0\}$

$$\iiint_E f(x, y, z)dV \ =$$

Hint for Q7 : 1,2,3,4,5,6,7

8. **One Variable Integrals** *(8 points)*

Calculate the following improper integral using multivariable calculus:

$$\int_{-\infty}^{+\infty} e^{-x^2}\ dx$$

Formulas

1. **Coordinate Systems**

Polar

$$\begin{cases} x = r\cos(\theta) \\ y = r\sin(\theta) \end{cases}$$

Cylindrical

$$\begin{cases} x = r\cos(\theta) \\ y = r\sin(\theta) \\ z = z \end{cases}$$

Spherical

$$\begin{cases} x = \rho\sin(\varphi)\cos(\theta) \\ y = \rho\sin(\varphi)\sin(\theta) \\ z = \rho\cos(\varphi) \end{cases}$$

2. **Surface Area**

$$Area(S) = \iint_D \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}\, dA$$ where S is the surface with equation $z = f(x, y)$.

3. **Jacobian Determinants**

2 dimensional

Rectangular: $J(x, y) = 1$

Polar: $J(r, \theta) = r$

3 dimensional

Rectangular: $J(x, y, z) = 1$

Cylindrical: $J(r, \theta, z) = r$

Spherical: $J(\rho, \theta, \varphi) = \rho^2 \sin(\varphi)$

Table of Integrals (Note : $C = 0$)	
$\int x^n dx = \frac{x^{n+1}}{n+1}$ for $n \neq 1$	$\int \frac{1}{x} dx = \ln\lvert x\rvert$
$\int e^x dx = e^x$	$\int a^x dx = \frac{a^x}{\ln a}$
$\int \ln(x) dx = x\ln(x) - x$	$\int \frac{\ln(x)}{x} dx = \frac{1}{2}(\ln(x))^2$
$\int \sin(x) dx = -\cos(x)$	$\int \cos(x) dx = \sin(x)$
$\int \sec(x)\tan(x) dx = \sec(x)$	$\int \csc(x)\cot(x) dx = -\csc(x)$
$\int \sin^2(x) dx = \frac{1}{2}x - \frac{1}{4}\sin(2x)$	$\int \cos^2(x) dx = \frac{1}{2}x + \frac{1}{4}\sin(2x)$
$\int \sec(x) dx = \ln\lvert\sec(x) + \tan(x)\rvert$	$\int \csc(x) dx = \ln\lvert\csc(x) - \cot(x)\rvert$
$\int \sec^2(x) dx = \tan(x)$	$\int \csc^2(x) dx = -\cot(x)$
$\int \sec^3(x) dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln\lvert\sec(x) + \tan(x)\rvert$	$\int \csc^3(x) dx = -\frac{1}{2}\csc(x)\cot(x) + \frac{1}{2}\ln\lvert\csc(x) - \cot(x)\rvert$
$\int \tan(x) dx = \ln\lvert\sec(x)\rvert$	$\int \cot(x) dx = \ln\lvert\sin(x)\rvert$
$\int \sinh(x) dx = \cosh(x)$	$\int \cosh(x) dx = \sinh(x)$
$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\arctan\left(\frac{x}{a}\right)$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\left(\frac{x}{a}\right)$
$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a}\ln\left\lvert\frac{x-a}{x+a}\right\rvert$	$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\lvert x + \sqrt{x^2 \pm a^2}\rvert$