University of Connecticut
Department of Mathematics

Math 210Q | Exam II: Derivatives | V0 - FS - 2006

Name: | Section: 002

Read This First !

- In a question with several parts, using an incorrect answer from a previous part correctly will get you full credit for that part, unless the wrong answer gravely simplifies the question.
- Except for question 8 (error analysis), leave numbers in the form π, e or $\sqrt{2}$ rather than "going numeric" with them.
- Use of a pocket calculator or graphing calculator is only allowed if the calculator cannot perform alpha numeric calculations. Storing of any formulas in memory is forbidden.
- In the very first question, take the limit along the x axis and the y axis and you'll see that they are different.
- If part of the solution is written outside the space provided for this solution clearly indicate this.
- The last page contains a few formulas. Look at it before you start!
- There is 5% extra credit in here. Good Luck! ☺

Grading - For Administrative Use Only

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1. **Limits** *(8 points)*

   Calculate the following limit if it exists, and if not clearly state why it does not exist. Simply writing "exists" or "does not exist" will not get you any credit.

   $$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x^2-2y^2} =$$

2. **Chain Rule** *(8 points)*

   Take $f : \mathbb{R}^3 \to \mathbb{R} : (x, y, z) \mapsto x^2 + y^2 + z^2$ where $x(t) = \cos(t)$, $y(t) = \sin(t)$ and $z(t) = 2t$. Calculate $f'(t)$. Your end result for $f'(t)$ should be a - rather simple - function expressed in terms of t only.

   $f'(t) =$

3. **Implicit Differentiation** *(8 points)*

Let x and y be connected by $x^2 + e^{xy} + y^2 = \pi^2$. Calculate $\frac{dy}{dx}$.

$\frac{dy}{dx} =$

4. **Higher Order Derivatives** *(8 points)*

Define $\Psi : \mathbb{R}^2 \to \mathbb{R} : (x, y) \mapsto e^x \sin(y)$. Calculate $\Delta\Psi$, the Laplacian of Ψ.

$\Delta\Psi =$

5. **Gradient and Directional Derivative** *(24 points)*

Consider $f : \mathbb{R}^2 \to \mathbb{R} : (x, y) \mapsto xe^y + \sin(x)$ and take $P = (\frac{\pi}{3}, 0)$.

(a) Calculate the gradient at P.

(b) Calculate the directional derivative at P *in the direction of* $< 1, 1 >$.

(c) Calculate the maximum rate of change at P. In which direction does this occur?

(d) Write down the total differential at P.

6. **Tangent Plane and Linearization** *(12 points)*

Take $f : \mathbb{R}^2 \to \mathbb{R} : (x, y) \mapsto x^2 + \sin(x) + y^2$

(a) Determine the equation of the tangent plane at $(0, 1)$

(b) Determine the linearization $\mathcal{L}(x, y)$ of $f(x, y)$ at $(\pi, 2)$. Note that this is not the same point as in part (a). However, you can recycle most of your calculations.

7. **Critical Points** *(12 points)*

Consider $f : \mathbb{R}^2 \to \mathbb{R} : (x, y) \mapsto x^3 + y^2 - 6x$

(a) Find both critical points.

$(x, y) \ =$

$(x, y) \ =$

(b) Use the second derivative test to determine the nature of these critical points.

(,) is a

(,) is a

8. **Maximum Error** *(9 points)*

A funky potential is defined by $\Phi(x, y) = x^2 + 2\pi x - \pi^2 e^y$. Assume that on both x and y there is a measuring error of 0.2. Calculate the error on $\Phi(\frac{3}{2}, 0)$ *using the total derivative.*

$\Phi\left(\frac{3}{2}, 0\right) = 1.8052\pm$

9. **Lagrange Multipliers** *(8 points)*

Maximize f by determining x and y:

$$\begin{cases} f(x, y) = xy \\ x + y = 2 \end{cases}$$

$(x, y) \ =$

10. **Lagrange Multipliers** *(8 points)*
Maximize f by determining x and y:

$$\begin{cases} f(x,y) = \sqrt{x(y+1)} \\ x + y = \pi \end{cases}$$

$(x, y) \;=$

Derivatives

$$f : \mathbb{R}^n \to \mathbb{R} : (x_1, \ldots, x_n) \mapsto f(x_1, \ldots, x_n)$$

Vector Notation

(i) *Partial Derivatives*
$$\frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n}$$

(ii) *Gradient*
$$\overrightarrow{\nabla f} = \left\langle \frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n} \right\rangle$$

(iii) *Total Differential*
$$df = \overrightarrow{\nabla f} \cdot \overrightarrow{dx}$$

(iv) *Chain Rule*
$$\frac{df}{dt} = \overrightarrow{\nabla f} \cdot \frac{d\overrightarrow{x}}{dt}$$

(v) *Directional Derivative*
$$D_{\overrightarrow{u}} f = \overrightarrow{\nabla f} \cdot \overrightarrow{u}$$

Note: Vector notation is the same for all dimensions. Putting $n = 2$ or $n = 3$ in the general scalar notation gives the expressions for $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively.

Scalar Notation

(i) *Partial Derivatives*
$$\frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n}$$

(ii) *Gradient*
$$\overrightarrow{\nabla f} = \sum_{k=1}^{n} \frac{\partial f}{\partial x_k} \overrightarrow{e_k}$$

(iii) *Total Differential*
$$df = \sum_{k=1}^{n} \frac{\partial f}{\partial x_k} dx_k$$

(iv) *Chain Rule*
$$\frac{df}{dt} = \sum_{k=1}^{n} \frac{\partial f}{\partial x_k} \frac{dx_k}{dt}$$

(v) *Directional Derivative*
$$D_{\overrightarrow{u}} f = \sum_{k=1}^{n} \frac{\partial f}{\partial x_k} u_k$$

$$f : \mathbb{R}^2 \to \mathbb{R} : (x, y) \mapsto f(x, y)$$

Scalar Notation

(i) *Partial Derivatives*
$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$$

(ii) *Gradient*
$$\overrightarrow{\nabla f} = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$$

(iii) *Total Differential*
$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

(iv) *Chain Rule*
$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

(v) *Directional Derivative*
$$D_{\overrightarrow{u}} f = \frac{\partial f}{\partial x} u_x + \frac{\partial f}{\partial y} u_y$$

$$f : \mathbb{R}^3 \to \mathbb{R} : (x, y, z) \mapsto f(x, y, z)$$

Scalar Notation

(i) *Partial Derivatives*
$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$$

(ii) *Gradient*
$$\overrightarrow{\nabla f} = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$$

(iii) *Total Differential*
$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz$$

(iv) *Chain Rule*
$$\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial z}\frac{dz}{dt}$$

(v) *Directional Derivative*
$$D_{\overrightarrow{u}} f = \frac{\partial f}{\partial x} u_x + \frac{\partial f}{\partial y} u_y + \frac{\partial f}{\partial z} u_z$$