University of Connecticut
Department of Mathematics

Math 210Q | Exam I: Vectors and Vector Functions | V0 - FS - 2006

Name: | Section: 002

Read This First !

- Use of a pocket calculator or graphing calculator is only allowed if the calculator cannot perform alpha numeric calculations. Storing of any formulas in memory is forbidden.
- There are no trick questions in this exam. Almost half of the questions have one line answers to accommodate the time constraint, not because there is some deeper hidden problem.
- If part of the solution is written outside the space provided for this solution clearly indicate this.
- The last page contains a few formulas. Look at it before you start!
- The answer to question four, part c is nine.
- There is 5% extra credit in here. Good Luck! ☺

Grading - For Administrative Use Only

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1. **Vectors** *(15 points)*

   (a) Calculate $< 1, 1, 1 > + < 2, -1, 3 >$.

   (b) Find the unit vector in the direction of $< 3, 2, 1 >$.

   (c) Find the vector of length 4 in the direction of $< 3, 2, 1 >$.

   (d) Calculate the angle between $< 3, 2, 1 >$ and $< 1, 2, -7 >$.

   (e) Calculate $< 3, 2, 1 > \cdot (< 1, 2, -7 > + < 1, 2, 3 >)$.

2. **Cross Product** *(10 points)*

   Simplify the following equations. Do not plug in numbers for $\overrightarrow{a}$ and $\overrightarrow{b}$, derive these in general. Remember the correct notation: b is a number and $\overrightarrow{b}$ is a vector.

   (a) $\overrightarrow{a} \times \overrightarrow{b} + \overrightarrow{b} \times \overrightarrow{a} =$

   (b) $\|\overrightarrow{a} \times \overrightarrow{b}\|^2 + (\overrightarrow{a} \cdot \overrightarrow{b})^2 =$

3. **Lines** *(5 points)*

   Give the equation (any form) of the line through the points $(1, 2, 3)$ and $(3, -2, 1)$.

4. **Lines and Planes** *(15 points)*

   (a) Find the point that is the intersection of the plane $x - 2y + z + 1 = 0$ and the line $< 0, 0, 7 > + < 4, -1, 2 > t$.

(b) Find the line that is the intersection of the planes $x + 5y + z = 0$ and $3x + 2y + z = 0$.

(c) Calculate the distance between the planes $2x - y + 2z - 9 = 0$ and $2x - y + 2z + 18 = 0$.

5. **Quadric Surfaces** *(15 points)*

   (a) Use traces to determine the type of the quadric surface $4x^2 + 3y^2 + 2z^2 = 1$.

   (b) Determine both possible values of K in $< x, y, z >=< 1, -1, K > t$ such that this line is part of the cone $z^2 = 2x^2 + 7y^2$.

   (c) Determine the center and the radius of the sphere with equation $x^2 - 2x + y^2 + z^2 = 3$

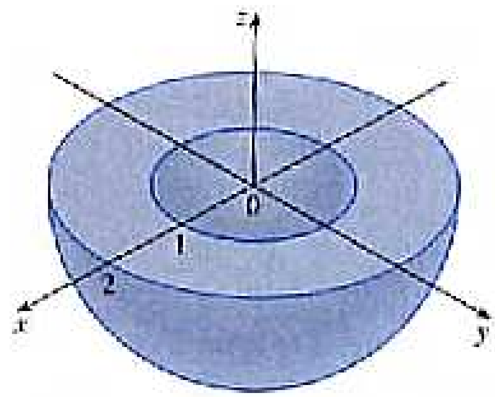
6. **Spherical and Cylindrical Coordinates** *(15 points)*

(a) Rewrite the One Sheet Hyperboloid $x^2 + y^2 - 2z^2 = 1$ in cylindrical coordinates.

(b) Write the inequalities that represent the region consisting of the points P that satisfy all three of the following restrictions:

- P is outside a sphere with radius 1.
- P is inside a sphere with radius 2.
- P is below the XY plane.

In other words, the southern hemisphere of a hollow ball. Use $<$ or $\leq$ for your inequalities, both will be considered correct.



(i) In Spherical Coordinates:

(ii) In Rectangular Coordinates:

(iii) In Cylindrical Coordinates:

7. **Vector Functions** *(15 points)*

Consider the curve $\overrightarrow{r} : \mathbb{R} \to \mathbb{R}^3 : t \mapsto < \cos(3t), \sin(3t), t^2 >$

(a) Calculate the velocity $\overrightarrow{v}$ and the acceleration $\overrightarrow{a}$.

(b) Calculate the following derivative.

$$\frac{d}{dt}[< t, 0, 0 > \cdot \overrightarrow{r}(t)] =$$

(c) Calculate the following derivative. If you want, you can write the solution in terms of $\overrightarrow{r}(t)$, $\overrightarrow{v}(t)$ and $\overrightarrow{a}(t)$.

$$\frac{d}{dt}[t\overrightarrow{r}(t)] =$$

8. **Arc Length** *(10 points)*

   Calculate the arc length of the curve with function $\overrightarrow{r}(t) =< \cos(3t), \sin(3t), \frac{2}{3}t\sqrt{t} >$ from the point where $t = -9$ to the point where $t = 0$.

9. **Tangent Vectors and Curvature** *(5 points)*

   Calculate the unit tangent vector $\overrightarrow{T}$ of the curve with function $\overrightarrow{r}(t) =< \ln(t), t^2, 1 >$

Formulas

Vectors and Norm	**Cross Product**
$\|\lambda \overrightarrow{a}\| = \|\lambda\| \, \|\overrightarrow{a}\|$ $\|\overrightarrow{a} + \overrightarrow{b}\| \leq \|\overrightarrow{a}\| + \|\overrightarrow{b}\|$ $\overrightarrow{a} \cdot \overrightarrow{b} = \|\overrightarrow{a}\| \, \|\overrightarrow{b}\| \cos(\vartheta)$ $\|\overrightarrow{a} \times \overrightarrow{b}\| = \|\overrightarrow{a}\| \, \|\overrightarrow{b}\| \sin(\vartheta)$	$\overrightarrow{a} \times \overrightarrow{b} = \begin{vmatrix} \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ $= < a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 >$
Cylindrical Coordinates	**Spherical Coordinates**
$\begin{cases} x = r\cos(\varphi) \\ y = r\sin(\varphi) \\ z = z \end{cases}$	$\begin{cases} x = \rho\sin(\varphi)\cos(\theta) \\ y = \rho\sin(\varphi)\sin(\theta) \\ z = \rho\cos(\varphi) \end{cases}$
Arc Length	**Curvature and Tangent Vector**
$L(\overrightarrow{r}; a, b) = \int_a^b \|\overrightarrow{r}\,'(t)\| dt$	$\kappa(t) = \frac{\|\overrightarrow{T}\,'(t)\|}{\|\overrightarrow{r}\,'(t)\|}$ where $\overrightarrow{T}(t) = \frac{\overrightarrow{r}\,'(t)}{\|\overrightarrow{r}\,'(t)\|}$