



*University of Connecticut
Department of Mathematics*

Math 116Q

Computer Algebra System Project

V0 - FS - 2005

Name:

Section: 005

Read This First !

- This Project is due on Friday, December 9th right after class. This is a hard deadline, because I don't want you to be spending any time on this during your reading days or exam week.
- Simplify your answers as much as possible. Maple often leaves answers in rather difficult to read form, even after using the `simplify` command. Never leave something like $\ln(e)$.
- Before you start typing away, load the "plots" package, i.e. type `with(plots)`; and hit return.
- In the exercises the commands needed are indicated. In some cases general commands like `simplify`, `evalf` and `assume` will prove useful.

Grading - For Administrative Use Only

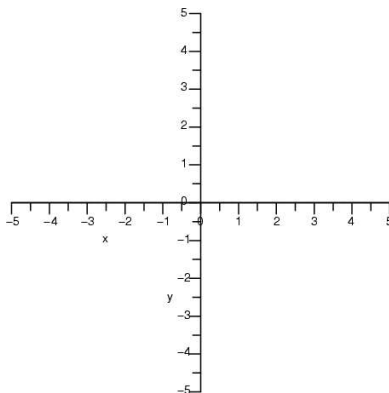
| GRADES | |
|-------------------------|------|
| Plotting Functions | /15 |
| Solving Equations | /15 |
| Calculating Limits | /15 |
| Calculating Derivatives | /15 |
| Calculating Integrals | /10 |
| Infinite Sums | /15 |
| Fourier Analysis | /10 |
| Working with Curves | /10 |
| Total | /100 |

1. **Plotting Functions***(15 points)*

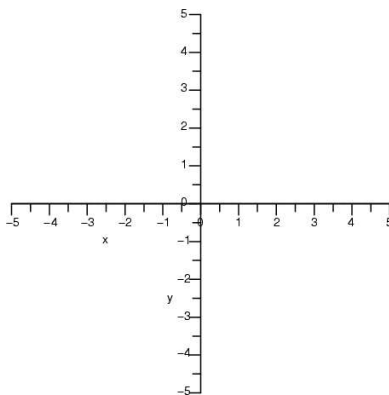
Use maple to plot the following functions. Make sure that you do appropriate rescaling before making a sketch. Clearly indicate asymptotes or endpoints on your graph; this will require you to think a little, not just use the computer.

Commands Needed: `plot`, `polarplot`

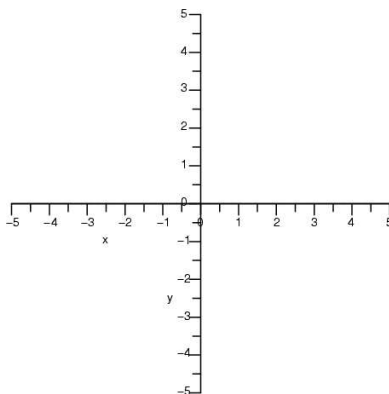
(a) $f : \mathbb{R} \rightarrow \mathbb{R} : x \mapsto e^x \ln(x)$



(b) $f : \mathbb{R} \rightarrow \mathbb{R}^2 : t \mapsto (t^2, 4 \cos(10t))$



(c) $f : [0, 2\pi] \rightarrow \mathbb{R} : \theta \mapsto r = 3 \sin(4\theta + \frac{\pi}{2}) + 1$



2. Solving Equations

(15 points)

Commands Needed: `solve`, `fsolve`, `evalf`, `plot`

- (a) Give all solutions of $x^3 + 2x^2 - 5x - 6 = 0$.
- (b) Solve $X = v_0 \cdot t + \frac{1}{2}at^2$ for t where t has to be positive.
- (c) Which is the only real solution of $5x^5 + 4x^4 + 3x^3 + 2x^2 + x = 1$? Give your answer as a numerical approximation with 6 significant figures.
- (d) Give **two** solutions of the equation $\cos(2x) + \sin(3x) = 0$
- (e) Solve the equation $\sin(x) = \frac{1}{4}x$. Give your answer as a numerical approximation with 6 significant figures. Warning: there are 3 solutions of which $x = 0$ is only one.

3. Calculating Limits*(15 points)*Command Needed: `limit`

$$(a) \lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x^4} =$$

$$(b) \lim_{x \rightarrow \infty} x \sin\left(\frac{1}{x}\right) =$$

$$(c) \lim_{x \rightarrow \infty} \ln(x) + \frac{1}{x} =$$

$$(d) \lim_{x \rightarrow 0^+} \ln(x) + \frac{1}{x} =$$

$$(e) \lim_{x \rightarrow 0^-} \ln(x) + \frac{1}{x} =$$

4. Calculating Derivatives

(15 points)

Command Needed: `diff`, `plot`

(a)
$$\frac{d}{dx} \left[\frac{\sin(x)}{\sqrt{x}} \right] =$$

(b)
$$\frac{d}{dx} \ln(\cos(\arctan(x^2 + 1))) =$$

(c)
$$\frac{d}{dx} e^{kx} \cos(nx) =$$
 (with $k, n \in \mathbb{N}$)

(d)
$$\frac{d}{dx} |x \sin(x)| =$$

What happens at $x = 0, \pm\pi, \pm2\pi, \pm3\pi, \dots$?

5. Calculating Integrals*(10 points)*Command Needed: `int`

(a) Using Maple, calculate

$$\int \frac{x^2}{\sqrt{9-x^2}} dx =$$

(b) Now calculate this integral by hand using the substitution $x = \cos(\theta)$:

$$\int \frac{x^2}{\sqrt{9-x^2}} dx =$$

(c) Why are these answers different?

6. Infinite Sums

(15 points)

Command Needed: `sum`

$$(a) \sum_{k=1}^{\infty} \frac{1}{k^2 + k} =$$

$$(b) \sum_{k=N}^{\infty} \frac{1}{k^2 - k} =$$

$$(c) \sum_{k=1}^{\infty} \frac{x^k}{k} =$$

$$(d) \sum_{k=1}^{\infty} \frac{x^k}{k!} =$$

$$(e) 4 \cdot \sum_{k=1}^{\infty} \frac{\sin(k\frac{\pi}{2})}{k} =$$

This series is not readily solved by the `sum` command. You will have to use the `evalf` command to get an idea what the sum would be. Note that this is one of the series that we saw in class, namely when calculating the Fourier series of

$$f(x) = \begin{cases} -\frac{1}{\pi}x - 1 & \text{for } -\pi < x < 0 \\ -\frac{1}{\pi}x + 1 & \text{for } 0 < x < \pi \end{cases}$$

7. Fourier Series

(10 points)

Commands Needed: `assume`, `int`, `sum`, `plot`, `taylor`Let g be defined as $g(x) = \frac{x^3}{\pi}$ be defined over $[-\pi, \pi]$ and periodic on the rest of \mathbb{R} .

(a) Calculate the coefficients of the Fourier series:

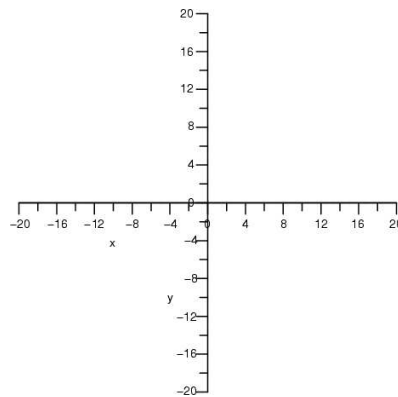
$$a_0 =$$

$$a_k =$$

$$b_k =$$

(b) Write the Fourier series:

$$\frac{x^3}{\pi} \sim$$

(c) Plot the partial sum F_{40} and sketch this on the grid below:(d) The other way around would be to approximate a sine function with a Taylor polynomial. So, calculate the first 6 terms (i.e. all terms that contain x^5 or lower) of the Taylor polynomial of $f(x) = \sin(2 * x)$ around $x = \pi$:

$$\sin(2x) \sim$$

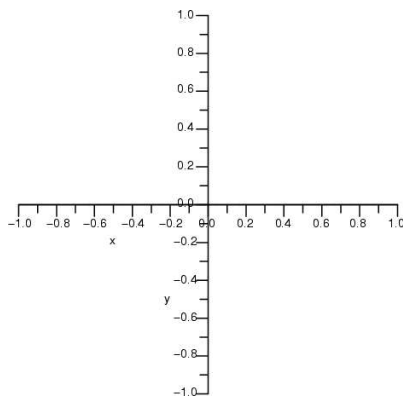
8. Working with Curves

(10 points)

Commands Needed: `polarplot`, `curves`Let f be a function defined in polar coordinates, i.e. $r = f(\theta)$:

$$f : [0, 4\pi] \rightarrow \mathbb{R} : \theta \mapsto \cos(\theta) \sin(3\theta)$$

- (a) Plot this curve. Make sure that you scale it right.



- (b) Calculate the area enclosed by this curve, i.e. first write the integral that you have to calculate and then calculate this integral using maple:

$$A =$$

- (c) Compare this value with the area of the unit circle. Especially if this does not appear realistic, type in the following:

```
animatecurve([cos(t)*sin(3*t),t,t=0..4*Pi],coords=polar,numpoints=100,frames=100)
```

Be patient. This requires a lot of calculations. Then Click on the empty XY -grid when it appears and then click on the "play" button in the upper left corner. Watch carefully. If you had to adapt your calculation of the area enclosed by this curve, write down what you changed: