Practice Final Solutions

$\boxed{\#1}$

The parameterization of C is $x(t) = 5\cos t$ and $y(t) = 5\sin t$ for $-\frac{\pi}{3} \le t \le \frac{\pi}{3}$ so we get

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} (25\cos^2 t + 25\sin^2 t)\sqrt{(-5\sin t)^2 + (5\cos t)^2}\, dt$$

$$= \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} 25 \cdot \sqrt{25}\, dt = 125\left(\frac{2\pi}{3}\right) = \frac{250\pi}{3} \blacksquare$$

$\boxed{\#2}$

$\int a(t)\,dt = \langle C_1, -5t + C_2, -2t + C_3 \rangle$ and by $V(0) = \langle 1, 5, 1 \rangle$ then $V(t) = \langle 1, -5t + 5, -2t + 1 \rangle$

$\int V(t)\,dt = \langle t + C_4, -\frac{5}{2}t^2 + 5t + C_5, -t^2 + t + C_6 \rangle$ and

by $r(0) = \langle 3, 3, 15 \rangle$ then $r(t) = \langle t+3, -\frac{5}{2}t^2 + 5t + 3, -t^2 + t + 15 \rangle \blacksquare$

$\boxed{\#3}$

$$\iint_D x\,dA = \int_0^2 \int_0^{x^2} x\,dy\,dx + \int_{-1}^{0} \int_{x^2}^{-x} x\,dy\,dx$$

$$= \int_0^2 x^3\,dx + \int_{-1}^{0} -x^2 - x^3\,dx = \left[\frac{x^4}{4}\Big|_0^2\right] + \left[-\frac{x^3}{3} - \frac{x^4}{4}\Big|_{-1}^{0}\right]$$

$$= \frac{16}{4} - \frac{1}{3} + \frac{1}{4} = \frac{47}{12} \blacksquare$$

$\boxed{\#4}$ The cylinder is a parametric surface for cylindrical coord

$$x(\theta, y) = 3\cos\theta \qquad r_\theta = -3\sin\theta\, i + 3\cos\theta\, k$$

$$z(\theta, y) = 3\sin\theta \quad \text{so} \quad r_y = j$$

$$y(\theta, y) = y$$

$$\text{so } |r_\theta \times r_y| = \left|\begin{matrix} i & j & k \\ -3\sin\theta & 0 & 3\cos\theta \\ 0 & 1 & 0 \end{matrix}\right| = |\langle 3\cos\theta, 0, -3\sin\theta\rangle| = 3$$

$$\text{so } \iint_S z\, dS = \int_0^{2\pi}\int_0^{6} 3\sin\theta \cdot 3\, dy\, d\theta = 9 \cdot 6 \int_0^{2\pi} \sin\theta\, d\theta$$

$$= 54\left[\cos\theta \Big|_0^{2\pi}\right] = 0 \blacksquare$$

$\boxed{\#5}$

$\boxed{A}$ The given line has $v = \langle 6, 3, 3\rangle$ so our line will be

$$\frac{x}{6} = \frac{y-1}{3} = \frac{z-3}{3} \blacksquare$$

$\boxed{B}$ Using the first 2 equations $\begin{matrix} 6s = 3t + 11 \\ 3s + 1 = t \end{matrix}$

$$\begin{matrix} 6s - 3t = 11 \\ 3s - t = -1 \end{matrix} \longrightarrow \begin{matrix} 6s - 3t = 11 \\ 6s - 2t = -2 \end{matrix} \quad \text{so} \quad -t = 13 \text{ so } t = -13$$

and $s = -\frac{14}{3}$.

Using the z equations $3s + 3 = -11$ and $t + 7 = -6$

so they're <u>skew</u> $\blacksquare$

C

$3(6s)+2(3s+1)+(3s+3)=1 \Rightarrow 18s+6s+2+3s+3-1=0$

so $27s=-4$ so $s=-\frac{4}{27}$ thus the point will

be $r\left(-\frac{4}{27}\right)=\left(-\frac{24}{27}, \frac{15}{27}, \frac{69}{27}\right)$ ■

#6

A $x_t=0$ so $f_y=-xz$ $\quad f_z=-xy+13$

$y_t=2 \quad z_t=\cos t$

so $F_t = -(2s)(\sin t)2 + \left(13-(2s)(2t-\cos s)\right)\cos t$ ■

B

$\frac{\partial z}{\partial y} = -\frac{-xz}{-xy+13} \qquad \frac{\partial z}{\partial x} = -\frac{3x^2-yz}{-xy+13}$ ■

C

$f_x = 3x^2-yz$ so $f_{xx}=6x$ so $f_{xxz}=0$ ■

#7

Note that C is negatively oriented so by Green's theorem

$$\oint_C 2y\,dx - 2x\,dy = -\int_0^1\int_0^\pi \left[(-2)-(2)\right] r\,d\theta\,dr$$

$$= 4\int_0^1\int_0^\pi r\,d\theta\,dr = 4\pi\left[\frac{r^2}{2}\right]_0^1 = 2\pi$$ ■

#8

Using cylindrical coordinates we get $\int_{-5}^{4}\int_{0}^{4}\int_{0}^{2\pi} r \cdot r \, d\theta \, dr \, dz$

$= 2\pi \cdot (4 - -5) \cdot \left[\frac{r^3}{3} \Big|_0^4 \right] = 18\pi \cdot \frac{64}{3} = 384\pi$ ■

#9

A

$a = \langle -1-1, 0-0, 0-1 \rangle = \langle -2, 0, -1 \rangle$
$b = \langle 2-1, 2-0, 2-1 \rangle = \langle 1, 2, 1 \rangle$

$n = a \times b = \begin{vmatrix} i & -j & k \\ -2 & 0 & -1 \\ 1 & 2 & 1 \end{vmatrix} = \langle 2, 1, -4 \rangle$ so we get

$2(x-1) + 1(y-0) - 4(z-1) = 0 \Rightarrow 2x - 2 + y - 4z + 4 = 0$

so $2x + y - 4z = -2$ ■

B

$n_1 = \langle 2, 1, -4 \rangle$
$n_2 = \langle 3, 3, -1 \rangle$ so $n_1 \times n_2 = \begin{vmatrix} i & -j & k \\ 2 & 1 & -4 \\ 3 & 3 & -1 \end{vmatrix} = \langle 11, -10, 3 \rangle$

we find the point by setting $x = 0$ and finding the point where both planes touch the yz plane simultaneously

$3y - z = 2$
$y - 4z = -2$ or $\begin{array}{r} 3y - z = 2 \\ 3y - 12z = -6 \\ \hline 11z = 8 \end{array}$ so $z = \frac{8}{11}$ so

$y = \frac{10}{11} \Rightarrow (0, \frac{10}{11}, \frac{8}{11})$ so

$x(t) = 11t$
$y(t) = -10t + \frac{10}{11}$ ■
$z(t) = 3t + \frac{8}{11}$

#10

A $\text{div } F = P_x + Q_y + R_z$

$$\text{curl } F = \nabla \times F = \begin{vmatrix} i & j & k \\ dx & dy & dz \\ P & Q & R \end{vmatrix} = (R_y - Q_z)i + (P_z - R_x)j + (Q_x - P_y)k \blacksquare$$

B

$\text{div } F = 0$

$P_y = 2y \qquad Q_x = 2 \qquad R_x = 1$

$P_z = 1 \qquad Q_z = -\sin z \qquad R_y = 0$

so $\text{curl } F = (\sin z)i + (0)k + (2-2y)k$

$= \langle \sin z, 0, 2-2y \rangle \blacksquare$

C

Not Conservative b/c curl $F \neq 0$ on all of $\mathbb{R}^3 \blacksquare$

#11

$$L = \int_0^1 \sqrt{(\sqrt{2})^2 + (e^t)^2 + (-e^{-t})^2}\, dt = \int_0^1 \sqrt{2 + e^{2t} + e^{-2t}}\, dt$$

$$= \int_0^1 \sqrt{(e^t + e^{-t})^2}\, dt = \int_0^1 e^t + e^{-t}\, dt = \left[e^t - e^{-t} \Big|_0^1 \right]$$

$$= e^1 - e^{-1} = e - \frac{1}{e} \blacksquare$$

#12

A $\nabla f = \langle 2x - 3x^2, -\sin y, -\frac{1}{z} \rangle \blacksquare$

$\boxed{B}$ $\nabla F(1,\pi,1) = \langle -1, 0, 1 \rangle$

$v = \langle -1, 0, 1 \rangle$ so $\frac{v}{|v|} = u = \langle -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \rangle$ gives us

$\langle -1, 0, 1 \rangle \cdot \langle -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \rangle = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$ $\blacksquare$

$\boxed{C}$ $\langle -1, 0, 1 \rangle$ is the direction of the gradient already so the maximum value is already found! $\sqrt{2}$ $\blacksquare$