Math 210Q-02 Spring 2006
Practice Final Exam

This version is condensed in spacing in order to save paper. Please have several other sheets of paper available to work on. This practice test is in no way all-inclusive or a guarantee that any of the material covered here will also appear on the actual final exam. This is for evaluation purposes only. The length, style, and content are highly reflective of what will be on the final exam though

1. Evaluate the integral $\int_C x^2 + y^2 \; ds$ where C is the arc of the circle of radius 5 centered at the origin that extends from -$\frac{\pi}{3}$ to $\frac{\pi}{3}$

2. Gonzo the Magnificent didn't learn his lesson the first time and he crawls into the human cannon once more. He is fired out of the big muzzle with an acceleration of $a(t) = -5j - 2k$. If he had initial velocity $v(0) = i + 5j + k$ and an initial position of $r(0) = 3i + 3j + 15k$. Find the equations for his velocity and position functions at time t.

3. Evaluate the integral $\iint_D x \; dA$ where D are the regions found between the curve $y = x^2$ and the function

$$y = \begin{cases} 0 & \text{if } 0 \leq x < \infty \\ -x & \text{if } -\infty < x \leq 0 \end{cases}$$

on the interval $[-1, 2]$

4. Evaluate the surface integral $\iint_S z \; dS$ where S is the surface of the the cylinder $x^2 + z^2 = 9$ that lies between $y = 0$ and $y = 6$

5. (a) Consider the line $r(t) =< 6t + 1, 3t - 2, 3t >$. Find the equation of a line, in symmetric form, that is parallel to r(t) and goes through the point (0, 1, 3)
   (b) Now consider your line and compare it to the line $r(t) =< 3t + 11, t, t + 7 >$. Are the lines parallel, intersecting, or skew?
   (c) Using the equation of your line, find the point where it intersects the plane 3x + 2y + z = 1

6. Let $f(x, y, z) = x^3 - xyz + 13z$ where $x(s, t) = 2s$, $y(s, t) = 2t - \cos s$, and $z(s, t) = \sin t$
   (a) Find the partial derivative of $f(x, y, z)$ with respect to t. You do not need to expand terms out.
   (b) Find the implicit derivatives of z with respect to y and x
   (c) Assuming that x, y, and z are independant variables (do not depend on s or t), find the equation of f_{xxz}

7. Evaluate the line integral $\int_C 2y\ dx - 2x\ dy$
where C is the line segment from $(1,0)$ to $(-1,0)$ and the arc of the circle $x^2+y^2=1$ from $(-1,0)$ to $(1,0)$

8. Evaluate $\iiint_E \sqrt{x^2+y^2}\ \ dV$ where E is the region that lies inside the cylinder $x^2+y^2 = 16$ and between the planes $z=4$ and $z=-5$

9. Consider the three, non-colinear points $(1,0,1)$, $(-1,0,0)$, and $(2,2,2)$

   (a) Find the equation of the plane that goes through all 3 points and put it into standard form.

   (b) Now consider the plane $3x+3y-z=2$. Find the equation of the line of intersection, in parametric form, that occurs when this plane intersects the one you found in part (a)

10. (a) Given a vector field F, state the definition of *curl F* and *div F*

   (b) Provided the vector field $F(x,y,z) = (y^2-z)i-(2x-\cos z)j+xk$, find the curl and divergence of the vector field

   (c) Is the vector field conservative? Give an explanation for your answer to support it.

11. Given the space curve $r(t) = \sqrt{2}ti + e^t j + e^{-t}k$, find the arc length over the interval $0 \leq t \leq 1$

12. Consider the equation $f(x,y,z) = x^2 - x^3 + \cos y - \ln z$

   (a) Find the equation of the gradient of $f(x,y,z)$

   (b) Find the derivative of $f(x,y,z)$ at the point $(1,\pi,-1)$ in the direction of the vector $< -1, 0, 1 >$

   (c) What is the value of the largest rate of change, in any direction, that can occur at the point $(1,\pi,1)$. Provide a supporting argument for your statement.