Math 2410, Quiz 8 (4/13/09) **Name:**

There are 2 questions. All answers have to be accompanied by supporting calculation or reasoning.

(1) Use the Laplace transform to solve

$$y' + y = e^{2t}$$

with initial conditions $y(0) = 2$.

Answer: Taking Laplace transform on both sides, we obtain

$$s\mathcal{L}[y] - 2 + \mathcal{L}[y] = \frac{1}{s-2} ,$$

which simplifies to

$$\begin{aligned}
\mathcal{L}[y] &= \frac{1}{(s+1)(s-2)} + \frac{2}{s+1} \\
&= -\frac{1}{3}\frac{1}{s+1} + \frac{1}{3}\frac{1}{s-2} + \frac{2}{s+1} \\
&= \frac{5}{3}\frac{1}{s+1} + \frac{1}{3}\frac{1}{s-2} .
\end{aligned}$$

On taking the inverse Laplace transform,

$$y = \frac{5}{3}e^{-t} + \frac{1}{3}e^{2t} .$$

(2) Use the method of undetermined coefficients to solve question 1. Check that the answers agree.

Answer: The complimentary solution to $y_c' + y_c = 0$ is $y_c = C_1 e^{-t}$. Let a particular solution be of the form $y_p = Ae^{2t}$. Putting into the non-homogeneous equation, we have

$$A(2+1)e^{2t} = e^{2t} .$$

Hence $A = 1/3$. The general solution is therefore $y = y_c + y_p = C_1 e^{-t} + e^{2t}/3$. Setting $y(0) = 2$, we find $C_1 = 5/3$. Thus the solution is:

$$y = \frac{5}{3}e^{-t} + \frac{1}{3}e^{2t} ,$$

which agrees with the answer in question 1.