Math 2410, Answer to Quiz 4 (2/23/09)

There are 2 questions. All answers have to be accompanied by supporting calculation or reasoning.

(1) By using an integrating factor, find the general solution to

$$\frac{dy}{dt} = -\frac{y}{1+t} + t^2$$

Answer: First, observe that the equation is a first order linear non-homogeneous equation and the coefficient of dy/dt is 1. When the equation is put in the standard form $dy/dt + a(t)y = g(t)$, we have $a(t) = 1/(1+t)$. The integrating factor η is given by:

$$\eta = e^{\int a(t)dt} = e^{\int \frac{dt}{1+t}} = e^{\log(1+t)} = 1 + t \ .$$

Hence multiplying the equation by $(1+t)$ on both sides, we have

$$(1+t)\{\frac{dy}{dt} + \frac{y}{1+t}\} = (1+t)t^2 \ .$$

Hence

$$\frac{d}{dt}[(1+t)y] = t^2 + t^3 \ .$$

Integrate the above equation to obtain:

$$(1+t)y = \frac{t^3}{3} + \frac{t^4}{4} + C$$

for any constant C. In other words,

$$y = \frac{1}{1+t}[\frac{t^3}{3} + \frac{t^4}{4} + C] \ .$$

(2) Find the general solution to the linear homogeneous equation $y'' - 6y' + 8y = 0$.

Answer: Let $y = e^{\alpha t}$. This will be a solution if $\alpha^2 - 6\alpha + 8 = 0$. Hence $\alpha = 4$ or $\alpha = 2$. In other words, both $y = e^{4t}$ and $y = e^{2t}$ are solutions.

Since the equation is linear and homogeneous, the general solution is

$$y = C_1 e^{4t} + C_2 e^{2t}$$

for some arbitrary constants C_1 and C_2.