Math 2410, Answer to Quiz 3 (2/16/09)

There are 2 questions. All answers have to be accompanied by supporting calculation or reasoning.

(1) Draw the bifurcation diagram for the equation

$$\frac{dy}{dt} = y^2(y+\mu) \ .$$

Fill in a few representative phase lines to indicate the stability of the equilibrium solutions. At what value of μ does bifurcation occur?

Answer: The equilibrium solution are solution of $y^2(y+\mu) = 0$. In other words, $y = 0$ or $y = -\mu$. In the (μ, y) plane, they can be represented by two straight lines $y = 0$ and $y = -\mu$. When $\mu = 0$, bifurcation occurs. The number of equilibrium solutions changes from 2 to 1 and back to 2 again. (See figure 1). □

(2a) Find the general solution to the linear homogeneous equation $\frac{dy}{dt} - y = 0$.

Answer: The general solution to the homogeneous equation is $y_h = Ce^t$ for any arbitrary constant C. □

(2b) Use the method of undetermined coefficient to find a particular solution to the linear non-homogeneous equation $dy/dt - y = 3e^{2t}$. Then find its general solution.

Answer: A particular solution will be of the form $y_p = Ae^{2t}$. Putting it into the non-homogeneous equation,

$$2Ae^{2t} - Ae^{2t} = 3e^{2t}$$

which gives $A = 3$. Thus the general solution for the non-homogeneous equation is:

$$y = y_p + y_h = 3e^{2t} + Ce^t$$

for any constant C. □

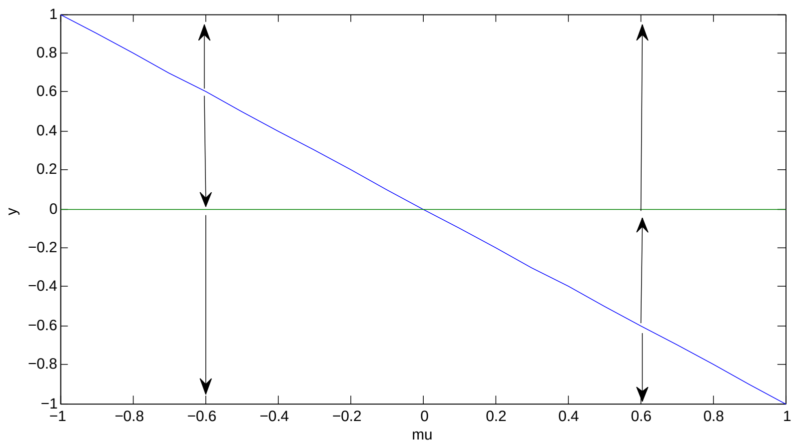

Figure 1: bifurcation diagram for $dy/dt = y^2(y + \mu)$.