Math 2410, Answer to Quiz 1 (2/2/09)

There are 2 questions. All answers have to be accompanied by supporting calculation or reasoning.

(1) Solve the initial value problem

$$\begin{cases} \dfrac{dy}{dt} = \dfrac{t^2}{y(1+t^3)} \ , \\ \\ y(0) = -2 \ . \end{cases}$$

You may need to use the method of substitution to integrate. Express y as a function of t in your final answer.

Answer: Integrating $y\ dy = \frac{t^2}{1+t^3} dt$, we have

$$\frac{y^2}{2} = \frac{1}{3} \log|1+t^3| + C \ .$$

To satisfy the initial condition, we require $C = 2$. Hence $y^2 = \frac{2}{3}\log|1+t^3|+4$. Consequently,

$$y = -\sqrt{\frac{2}{3}\log|1+t^3|+4} \ .$$

Note that we select the negative sign for square root, because we know that $y(0) = -2$. □

(2) Consider the equation $dy/dt = (y-1)(y-2)$.

(a) Find all its equilibrium solutions.

Answer: Setting $dy/dt = 0$ to find the equilibrium solutions, we have $(y-1)(y-2) = 0$. Thus $y = 1$ and $y = 2$ are the equilibrium solutions. □

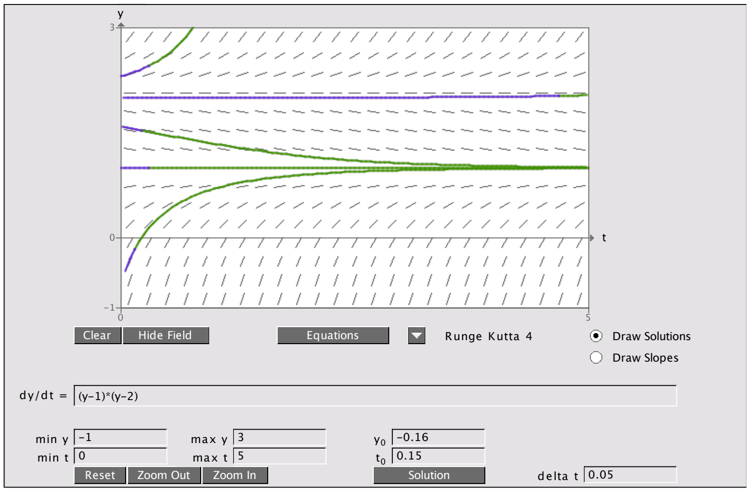

Figure 1: slope field for $dy/dt = (y-1)(y-2)$

(b) Draw the slope field and a few representative solution trajectories representing different initial conditions in the (t, y) plane.

Answer: First draw the lines $y = 1$ and $y = 2$, which represent the two equilibrium solutions. Then for the solution curves that start with initial conditions $y(0) > 2$, $1 < y(0) < 2$ and $y(0) < 1$, respectively. The plot is given in figure 1. □