Risk-Based Valuation
Statement of the problem

• The CAPM-based capital budgeting theory does not work for capital-constrained firms
  ○ Idiosyncratic risk may be costly
  ○ Cash flows in different periods may be correlated
    ▷ (this matters if idiosyncratic risk matters)
  ○ Benchmarks and comparables should be used when available
  ○ Using forwards in a CAPM context can be challenging
  ○ Options and other nonlinear relationships are difficult to include
  ○ Some CAPM parameters are unknown
    ▷ (e.g. correlation between project and market return)
  ○ Project data normally occur in prices and levels, not returns

• Firms lack an integrated and consistent framework for valuing projects in capital-constrained environments.
  ○ This presentation uses simulation as a unifying framework to achieve this objective
Outline of this presentation

- Develop consistent framework for CAPM, forwards & options
- General valuation equation for any given risk measure
  - Risk-neutrality and time-neutrality
- Derive pricing formulae with idiosyncratic risk
- Explain the derivation of the cost of risk
- Describe integrated valuation framework
Simplest case: Obtaining the CAPM

- One-year project
- Simulated levels of cash flow ($C_1$) and market index ($M_1$)
- Regress $C_1$ on $M_1$
  - $C_1 = [\mu_C - \beta_L \mu_M] + \beta_L M_1 + \epsilon$
  - (note $\beta_L$ is in levels not returns; $\beta_L = \text{cov}(C_1, M_1) / \text{var}(M_1)$)
- Discount risk-free and market-correlated cash flows assuming residual risk is unpriced
  - $V_0 = [\mu_C - \beta_L \mu_M] / (1 + r_f) + \beta_L M_0 + 0$
  - $V_0 = [\mu_C - \beta_L \{\mu_M - M_0(1 + r_f)\}] / (1 + r_f)$
  - $V_0 = \mu_C / (1 + r_f) - \beta_L \{\mu_M / (1 + r_f) - M_0\}$
- Substitute
  - $\beta_L = \beta V_0 / M_0$, $C_1 = V_0(1 + r_V)$, $M_1 = M_0(1 + r_M)$
  - $E(r_V) = r_f + \beta (E(r_M) - r_f)$
  - $V_0 = \mu_C / (1 + E(r_V))$

Equiv

- Replication pricing
- Risk-neutral pricing
- Time-neutral pricing

$\beta_L \approx \beta V_0 / M_0$, $C_1 \approx V_0(1 + r_V)$, $M_1 \approx M_0(1 + r_M)$

CAPM expected return eq.
Valuing a one-year oil project using forwards

- \( W = \text{WTI} \) (West Texas Intermediate Crude Oil)
- Simulated levels of cash flow \((C_1)\) and oil prices \((W_1)\)
  - Cash flow depends on revenues and costs, both of which are functions of oil prices
- Regress \( C_1 \) on \( W_1 \)
  - \( C_1 = [\mu_C - \beta_L \mu_W] + \beta_L W_1 + \epsilon \)
- Discount risk-free and oil-correlated cash flows assuming residual risk is unpriced and \( F_W = \) forward price of oil
  - \( V_0 = [\mu_C - \beta_L \mu_W]/(1+r_f) + \beta_L F_W/(1+r_f) \)  
    Replication
  - \( V_0 = [\mu_C - \beta_L \{\mu_W - F_W\}]/(1+r_f) \)  
    Risk-neutral
  - Equivalent to time-neutral valuation if \( F_W \Leftarrow F_W/(1+r_f) \)
- Q: What if the relationship between \( C \) and \( W \) is nonlinear?
Valuing an option in the Black-Scholes framework

- Slight difference: Allow only one rebalancing period, at time zero
- Simulated levels of stock price \( S_T \) and call option payout \( C_T \)
  - \( \alpha \) = continuous expected growth rate of the stock
- Regress \( C_T \) on \( S_T \)
  - \( C_T = a + bS_T \)
    - \( b = \frac{[E(C_T S_T) - E(C_T) E(S_T)]}{[E(S_T^2) - E(S_T)^2]} \)
    - \( a = E(C_T) - b E(S_T) \)
- Discount risk-free and stock-correlated cash flows assuming residual risk is unpriced
  - \( C_0 = S_0 \exp[(\alpha - r_f)T] N(d_1(\alpha)) - X \exp(-r_f T) N(d_2(\alpha)) - bS_0(\exp[\alpha - r_f T] - 1) \)
    - \( d_1(\alpha) \) is the Black-Scholes \( d_1 \) with \( \alpha \) substituted for \( r_f \)
  - Simplifies to Black-Scholes when \( \alpha = r_f \)
Every asset satisfies the general valuation equation GVE
- Expected return = Required return
- \{Exp capital gain\} + Exp cash flow = Cash opportunity cost + Risk compensation
- \{E[V_{t+1}] - V_t\} + E[C_{t+1}] = rV_t + kR_{t+1} \text{ at all times } t

In the CAPM example presented earlier, the risk compensation simplifies to
- \(kR_{t+1} = \beta(E(r_M) - r_f)V_0 = \beta_L[\mu_M - (1+r_f)M_0]\)

If we move the cost of risk \((kR_{t+1})\) to the left side of the GVE equation, we obtain risk-neutrality

The time-neutral transformation of cash flows is achieved by discounting all the cash flows and risk measures at the riskless rate and then using an effective riskless rate of 0.
- Test: Discount cash flows and risk measures at the riskless rate in the GVE and apply a zero discount rate
  - \(E[V_{t+1}]/(1+r) - V_t + E[C_{t+1}]/(1+r) = 0 + kR_{t+1}/(1+r)\)
  - Multiplying by \((1+r)\) and rearranging terms, this produces the original GVE
Pricing idiosyncratic risk

- Normally distributed cash flow $C_1$ in one year ($\mu_C, \sigma_C$)
- Apply the GVE:
  - $\{\mu_C - V_0\} + 0 = r_f V_0 + k\sigma_C$
  - $V_0 = [\mu_C - k\sigma_C] / (1 + r_f)$
- Expected return equation
  - $E(r_V) = r_f + k\sigma_C / V_0$
- Same cash flow, but now correlated with the market
- Apply the GVE:
  - $\{\mu_C - V_0\} + 0 = r_f V_0 + \beta_L[\mu_M - (1+r_f)M_0] + k\sigma_\varepsilon$
  - $V_0 = [\mu_C - \beta_L[\mu_M - (1+r_f)M_0] - k\sigma_\varepsilon] / (1 + r_f)$
- The expected return equation
  - $E(r_V) = r_f + \beta[E(r_M) - r_f] + k\sigma_\varepsilon / V_0$
Joint normally distributed cash flows $C_1, \ldots, C_N$ with correlation matrix $R$, standard deviation vector $\sigma$ and mean vector $\mu$

- The Cholesky decomposition of $R$ is given by $C$, and $I$ is the identity matrix

Make time-neutral conversion for convenience

- Convert $C_j^* = C_j/(1+r_f)^j$
- Replace $\mu_j^* = \mu_j/(1+r_f)^j$ and $\sigma_j^* = \sigma_j/(1+r_f)^j$

Choose risk measure and value

- Variance of total value (PVAR) $V_0 = \mu^*1 - kz (\sigma^* R \sigma^*)^{1/2}$
- Stdev of total value (RPV) $V_0 = \mu^*1 - kz (\sigma^* C 1)$
- Stdev of total value, zero corr (CFAR) $V_0 = \mu^*1 - kz (\sigma^* I 1)$
Properties of these models

- Idiosyncratic risk matters
  - hedging adds value
- Correlations between cash flow periods matter
- Ordering of cash flows matters
- Values are non-additive
  - a negative NPV incremental project can add value
- Easy to add market factors (multifactor risk)
- Easy to include option-like payoffs
Determining the private cost of risk (k)

- k is a measure of the adverse impact caused by increased risk
- If an agent accepts a contract or purchases an asset, the incremental risk will generally
  - Add to the risk of the agent’s cash flow
  - Increase the risk of declines in future wealth
  - Increase the likelihood of financial distress or bankruptcy
- The value of k is chosen on the margin so the agent is compensated for the cost to his income statement or balance sheet.
- Example
  - Suppose each additional $100,000 of risk increases the likelihood of financial distress by 5%, and the cost of financial distress is $250,000. In this case k = expected loss per dollar of risk = (5% of $250,000)/100,000 = 12.5%.
- Most financial institutions have determined an explicit cost of risk which they use in their valuations of financial assets and contracts.
The consistent framework

- The cash flows of a project along with its traded value drivers can be simulated.
  - Relationships may be linear or nonlinear.
- Time-neutralize cash flows, traded assets and forward prices.
- Regress adjusted cash flows on traded value drivers and compute covariance matrix of residuals.
- Choose the appropriate risk measure.
- Determine the appropriate cost of risk $k$.
- Value the project using PVAR or RPV.
- Replace NPV criterion:
  - Accept an incremental project if the risk-based valuation of the package exceeds the risk-based valuation of the standalone project.