Math 210Q Final Exam Review Sheet

1. If $\boldsymbol{v} = \boldsymbol{i} + 2\boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{w} = -2\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$, then find
   (a) $\boldsymbol{v} + 2\boldsymbol{w}$
   (b) $\boldsymbol{v} \cdot \boldsymbol{w}$
   (c) $|v|$ and $|w|$
   (d) the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$
   (e) the scalar projection of $\boldsymbol{v}$ onto $\boldsymbol{w}$
   (f) a unit vector orthogonal to both $\boldsymbol{v}$ and $\boldsymbol{w}$

2. Let $P = (1, 2, 3)$, $Q = (2, 4, 5)$, and $R = (-1, 3, 5)$.
   (a) Find parametric equations for the line through the points P and Q.
   (b) Find an equation of the plane through P, Q, and R.

3. A particle moves on the curve $\boldsymbol{r}(t) = (\ln t)\boldsymbol{i} + 2t\boldsymbol{j} + t^2\boldsymbol{k}$, where $t > 0$. Determine
   (a) the velocity, speed and acceleration at any time t.
   (b) the arc length from $t = 1$ to $t = 2$.
   (c) the unit tangent vector and the unit normal vector at $t = 1$.

4. Let $w = \ln(x^2 + y^2 + z^2)$, where $x = \sin(st)$, $y = \cos(st)$, and $z = s^2t^2$. Find $\dfrac{\partial w}{\partial s}$ and $\dfrac{\partial w}{\partial t}$ in terms of s and t.

5. Find and classify all the critical points of $f(x, y) = 12x + 27y - x^3 - y^3$.

6. Find the equation of the tangent plane to the surface $z = xe^y$ at the point where $x = 0$ and $y = 1$.

7. Find the directional derivative of $f(x, y, z) = x^y + x\sqrt{1 + z}$, at the point $(1, 2, 3)$ in the direction of the vector $\boldsymbol{v} = 2\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$.

8. Evaluate $\displaystyle\int\int_D e^{-x^2-y^2}\, dA$, where D is the region inside the unit circle $x^2 + y^2 = 1$.

9. Find the volume of the solid E which lies below the graph of $z = x^2 + y^2$, and above the region D on $xy-$plane, where D is bounded by $y = x^2$ and $x = y^2$.

10. Evaluate $\displaystyle\int\int\int_E \frac{1}{x^2 + y^2 + z^2}\, dV$, where E is the region between the graphs of $x^2 + y^2 + z^2 = 1, x^2 + y^2 + z^2 = 9$, and $z = \sqrt{3x^2 + 3y^2}$.

11. Calculate the work done by the force field $\boldsymbol{F}(x, y, z) = xy\boldsymbol{i} + x^2\boldsymbol{j}$ in moving a particle along the curve C described by the vector function $\boldsymbol{r}(t) =< \sin(t), 1 + t >$, $0 \le t \le \pi$.

12. Evaluate the line integral $\displaystyle\int_C x^3 z\, ds$ where C is the curve described by the vector function $\boldsymbol{r}(t) = \langle 2\sin(t), t, 2\cos(t)\rangle$, $0 \le t \le \pi/2$.

13. Evaluate the line integral $\int_C (e^x + xy)dx + (x + y^5 + \sin(y^2))dy$ where C is the rectangle with vertices $(0,0), (0,2), (2,4), (0,4)$, traversed counterclockwise.

14. Let $\boldsymbol{F}(x,y) = (x^2 + 3y\cos(x))\boldsymbol{i} + (3\sin(x) + y)\boldsymbol{j}$ and let C be the line segment from $(0,0)$ to $(1,2)$.

    (a) Compute $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$.

    (b) Suppose instead that we take the path C_1 consisting of the parabola $y = 2x^2$, with $0 \leq x \leq 1$. (Note that C and C_1 have the same initial and terminal points.) What would be the value of $\int_{C_1} \boldsymbol{F} \cdot d\boldsymbol{r}$?

15. Evaluate $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$, where $\boldsymbol{F}(x,y,z) = (3x^2 + yz)\boldsymbol{i} + (xz + 2z^3)\boldsymbol{j} + (2x^2z + 3yz^2)\boldsymbol{k}$, and C is the curve with vector equation $\boldsymbol{r}(t) = t^2\boldsymbol{i} + t^3\boldsymbol{j} + t\boldsymbol{k},\ 0 \leq t \leq 1$.

16. Let $f(x,y,z) = x^2yz$. Calculate the following:

    (a) ∇f

    (b) $\nabla \cdot (\nabla f)$

    (c) $\nabla \times (\nabla f)$

17. Let $\boldsymbol{F}(x,y,z) = \langle x^2 + y, yz, xy \rangle$. Calculate the following:

    (a) $\nabla \cdot \boldsymbol{F}$

    (b) $\nabla \times \boldsymbol{F}$

    (c) $\nabla(\nabla \cdot \boldsymbol{F})$

    (d) $\nabla \cdot (\nabla \times \boldsymbol{F})$

18. Let $\boldsymbol{G}(x,y,z) = x^2y\boldsymbol{i} + yz^2\boldsymbol{j} + y^2z\boldsymbol{k}$. Determine if $\boldsymbol{G}$ is conservative. If so, find a potential for $\boldsymbol{G}$.

19. Let $\boldsymbol{F}(x,y,z) = (y + ze^x)\boldsymbol{i} + (x + y)\boldsymbol{j} + (e^x - \cos(z))\boldsymbol{k}$. Determine if $\boldsymbol{F}$ is conservative. If so, find a potential for $\boldsymbol{F}$.

20. Find the area of the surface $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.