Math 210Q **Name:** _____
Practice Exam 2

Directions: Show all of your work and clearly indicate your answers. Correct answers with no work may not receive credit.

1. Find all second partial derivatives of the function $f(x, y) = xe^{-2y}$.

2. Find the gradient of the function $f(x, y, z) = z^2 e^{x\sqrt{y}}$ at the point $P = (0, 4, 3)$. Find the maximum rate of change of f at P. In which direction does it occur?

3. Let $g(x, y) = 2\sqrt{x} - y^2$ and let $P = (1, 5)$. Find the directional derivative of g at the given point in the direction of the vector $\boldsymbol{v} =< 4, 1 >$. Find the maximum rate of change of g at P. In which direction does it occur?

4. Find the equation of the tangent plane to the surface $z = e^x(\sin(y) + \cos(y))$ at the point where $x = 0$ and $y = \pi/2$.

5. Find the equation of the tangent plane to the surface $\sin(xyz) = x + 2y + 3z$ at the point $(2, -1, 0)$.

6. Find the points on the sphere $x^2 + y^2 + z^2 = 1$ where the tangent plane is parallel to the plane $2x + y - 3z = 2$.

7. Find the linear approximation of the function $f(x, y, z) = \frac{e^x}{y^2+z^2}$ at the point $(0, 1, 1)$ and use it to estimate the number $\frac{e^{0.02}}{(1.01)^2+(0.97)^2}$.

8. Suppose that $f(x, y, z) = x\sqrt{y} + \frac{z^2}{x}$, where $x = t^3 - 4t,\ y = e^{-2t},\ z = t^2 - 4$. Use the Chain Rule to find $\dfrac{\mathrm{d}f}{\mathrm{d}t}$.

9. Suppose that $w = x^2y + xy^2$, where $x = u^2 + v,\ y = u - v^2$. Use the Chain Rule to find $\dfrac{\partial w}{\partial u}$ and $\dfrac{\partial w}{\partial v}$.

10. Suppose $u(x, y) = e^{ax} \sin(ay)$, where a is a constant. Show that f is harmonic, that is,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$

11. If $z = y + f(x^2 - y^2)$, where f is differentiable, show that

$$y \frac{\partial z}{\partial x} + x \frac{\partial z}{\partial y} = x.$$

12. Find all the critical points of the function $f(x, y) = x^3 - 6xy + 8y^3$. Use the Second Derivatives Test to classify them as maxima, minima or saddle points.

13. Find all the critical points of the function $f(x, y) = (x^2+y)e^y$. Use the Second Derivatives Test to classify them as maxima, minima or saddle points.

14. Find the points on the surface $z^2 = xy + 1$ that are closest to the origin

15. If the length of the diagonal of a rectangular box must be L, what is the largest possible volume?

16. Find the absolute maximum and minimum values of f on the set D, where $f(x, y) = xy^2$ and $D = \{(x, y) : x^2 + y^2 \leq 3\}$.

17. Suppose f is a differentiable function of x and y, and $g(u, v) = f(ue^v, u^2 + \cos v)$, Use the table of values to calculate $g_u(1, 0)$ and $g_v(1, 0)$.

	f	g	f_x	f_y
$(1, 0)$	3	6	4	8
$(1, 2)$	6	3	2	5

18. Find all of the points (x, y) where the maximum rate of change of the function $f(x, y) = x^3 + y^2 + x - y + 32$ is in the same direction as the vector $\boldsymbol{v} =< 1, 1 >$.