Math 210Q **Name:** ________________________________

Practice Exam 1

Directions: Show all of your work and clearly indicate your answers. Correct answers with no work may not receive credit. **Good Luck!**

Question 1. (25 points) Let $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$.

(a) Find $\mathbf{a} \cdot \mathbf{b}$ and $\mathbf{a} \times \mathbf{b}$

(b) Determine the angle between $\mathbf{a}$ and $\mathbf{b}$.

(c) Find the unit vector in the direction of $\mathbf{a}$.

(d) Find parametric equations for the line in $\mathbf{R}^3$ that passes through the point $(0, 1, 0)$ in the direction of $\mathbf{a}$. Do the same in the direction of $\mathbf{b}$.

(e) Find the equation of the plane passing through $(1, 1, 1)$ and with normal vector $\mathbf{b} - \mathbf{a}$.

Question 2. (5 points) Find the area of the triangle with vertices $P(1, 4, 6)$, $Q(-2, 5, -1)$, and $R(1, -1, 1)$.

Question 3. (5 points) Suppose one angle of a parallelogram is $\pi/3$. If all the sides of the parallelogram have length 1, what is the area of the parallelogram?

Question 4. (5 points) Find the point in which the line with parametric equations

$$\begin{aligned} x &= t \\ y &= 3 - 2t \\ z &= 3 + 3t \end{aligned}$$

intersects the plane

$$z = 3x + 2y.$$

Question 5. (5 points) Find the parametric equation of the line described by the intersection of these planes.

$$3x - 6y - 2z = 15$$
$$y - 2z = 5$$

Question 6. (5 points) Find a vector perpendicular to the plane that passes through the points $P(1, 0, 1)$, $Q(0, 1, 0)$, and $R(0, 0, 1)$. What is the equation of the plane containing these points?

Question 7. (5 points) Consider the surface whose cartesian equation is given by

$$x^2 + z^2 = 9.$$

Sketch a graph of the equation above by drawing some traces.

Question 8. (5 points) Consider the quadric surface whose cartesian equation is given by

$$z^2 = x^2 + y^2.$$

Sketch a graph of the equation above by drawing some traces.

Question 9. (20 points) A particle in space moves according to the position function:

$$\mathbf{r}(t) = t\ \mathbf{i} + \sin(2t)\ \mathbf{j} + \cos(2t)\ \mathbf{k}.$$

1. Find formulas for the velocity, speed and acceleration at any time t.
2. Find the tangential and normal components of acceleration.
3. Find the unit normal vector $\mathbf{N}(t)$.

Question 10. (5 points) Suppose $\mathbf{r}(t)$ describes a curve. Show that the unit tangent vector, $\mathbf{T}(t)$, and unit normal vector, $\mathbf{N}(t)$, are orthogonal to each other.

Question 11. (5 points) Find the angles (approximately) of the triangle with vertices $A(1,0), B(3,6), C(-1,4)$.

Question 12. (5 points) Find a unit vector orthogonal to the vectors $\mathbf{i} - \mathbf{j} + \mathbf{k}$ and $4\mathbf{j} + 4\mathbf{k}$

Question 13. (5 points) True or false? The *cross product* of two unit vectors is also a unit vector.

Question 14. (10 points) In each of the problems below, determine weather the given lines are perpendicular, parallel, or neither:

1. $L_1 : x = 3 - t,\quad y = 1 + 2t,\quad z = 5 + 3t$
   $L_2 : x = 2s,\quad y = 4 - 4s,\quad z = 1 - 6s$
2. $L_1 : x = 3 - t,\quad y = 1 + 2t,\quad z = 5 + 3t$
   $L_2 : x = -3s,\quad y = 4 + 6s,\quad z = 1 - 5s$

Question 15. (5 points) Find the equation of the plane containing the vectors $< 1, 0, 1 >$ and $< 1, -2, 2 >$ and passing through the point $(1, 2, 3)$

Question 16. (5 points) Find the parametric equations of the line passing through the point $(2, 3, 1)$ and perpendicular to the plane $2x - y + 4z = 12$.

Question 17. (5 points) Find the parametric equations of a line passing through the point $(2, 0, 2)$ and *contained in* the plane $2x - y + 4z = 12$. [Think about it for a minute.]