Your name: ____________________

Quiz no. 7 (2110 Multivariable Calculus, Fall 2009)

December 6, 2009

20 min.

1. *(10 points)* Let C be the curve that consists of the line segment from $(-1,0)$ to $(1,0)$, and the parabola $y = 1 - x^2$ from $(1,0)$ to $(-1,0)$. Compute the integral

$$\oint_C y\,dx - x\,dy$$

   (a) directly, **and**

   (b) using Green's Theorem.

   Hint: As a parametrization of the parabola-part of C you can use $x =$ [illegible], $y = 2t - t^2$ for $0 \le t \le 2$.

   $1-t$

$+3yz^2$

2. *(5 points)* Given the vector field $F(x, y, z) = \langle y^2 z, 2xyz + z^3, xy^2 + \rangle$, find a function $f(x, y, z)$ such that $F = \nabla f$.

3. *(5 points)* Give a parametrization for the part of the sphere $x^2 + y^2 + z^2 = 9$ that lies in front of $x = 0$ (i.e., that part that has $x \geq 0$).

Your name: Sample solutions

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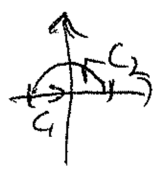
(a) 

$$\oint_C = \int_{C_1} + \int_{C_2}$$

C_1: $x = t,\ y = 0$ for $-1 \le t \le 1$; $\quad x'(t) = 1,\ y'(t) = 0$

$$\int_{C_1} y\,dx - x\,dy = \int_{-1}^{1} (0 \cdot 1 - t \cdot 0)\,dt = 0$$

C_2: $x = 1 - t,\ y = 2t - t^2$ for $0 \le t \le 2$; $\quad x'(t) = -1,\ y'(t) = 2 - 2t$

$$\int_{C_2} y\,dx - x\,dy = \int_0^2 [(2t - t^2)\cdot(-1) - (1-t)\cdot(2-2t)]\,dt = \int_0^2 (-2t + t^2 - 2 + 4t - 2t^2)\,dt$$

$$= \int_0^2 (-2 + 2t - t^2)\,dt = \left[-2t + t^2 - \frac{t^3}{3}\right]_0^2 = -4 + 4 - \frac{8}{3} = -\frac{8}{3}$$

(b)
$$\oint_C y\,dx - x\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA. \quad Q(x,y) = -x \quad P(x,y) = y$$

$$= \iint_D (-1 - 1)\,dA = \int_{-1}^{+1}\int_0^{1-x^2} (-2)\,dy\,dx = \int_{-1}^{1} (-2 + 2x^2)\,dx = \left[-2x + \frac{2x^3}{3}\right]_{-1}^{1} = -\frac{8}{3}$$

$+3zy^2\rangle$

2. *(5 points)* Given the vector field $F(x, y, z) = \langle y^2z, 2xyz+z^3, xy^2$ $\cancel{}$$\rangle$, find a function $f(x, y, z)$ such that $F = \nabla f$.

$f_x = y^2 z \Rightarrow f(x,y,z) = xy^2z + g(y,z)$

$f_y = 2xyz + z^3 \Rightarrow 2xyz + g_y(y,z) = 2xyz + z^3 \Rightarrow g_y(y,z) = yz^3 + h(z)$

$f_z = xy^2 + 3zy^2 \Rightarrow h(z) = C$

$f(x,y,z) = \cancel{xy^2z}\ xy^2z + yz^3 + C$

3. *(5 points)* Give a parametrization for the part of the sphere $x^2 + y^2 + z^2 = 9$ that lies in front of $x = 0$ (i.e., that part that has $x \geq 0$).

$x = \sin\phi\ \cancel{\cos\sin\theta}\ \cos\theta$

$y = \sin\phi\ \sin\theta$

$z = \cos\theta$