Your name: ________________

Quiz no. 3 (2110 Multivariable Calculus, Fall 2009)

October 2, 2009

20 min.

1. *(3 points)* Find the unit tangent vector $\vec{T}(t)$ of $\vec{r}(t) = \langle e^t, \cos t, t\rangle$ at the point $(1, 1, 0)$.

$\vec{r}\,'(t) = \langle e^t, -\sin t, 1\rangle$

$t=0$: $\vec{r}\,'(0) = \langle 1, 0, 1\rangle$ $\quad |\vec{r}\,'(0)| = \sqrt{1^2+0^2+1^2} = \sqrt{2}$

$\vec{T}(t) = \frac{\langle 1,0,1\rangle}{\sqrt{2}}$

2. *(8 points)* Consider the curve $\vec{r}(t) = \langle 4\sin t, 3t, 4\cos t\rangle$.

   (a) Find the arc length of $\vec{r}(t)$ for $0 \le t \le 2$.

   (b) Give an equation for the osculating plane at the point $(0, 3\pi, -4)$.

(a) $\vec{r}\,'(t) = \langle 4\cos t, 3, -4\sin t\rangle$

Arclength: $\int_0^2 \sqrt{(4\cos t)^2 + 3^2 + (-4\sin t)^2}\, dt$

$= \int_0^2 \sqrt{16(\cos^2 t + \sin^2 t) + 9}\, dt = \int_0^2 \sqrt{16+9}\, dt$

$= \int_0^2 5\, dt = 5t\Big|_0^2 = 10 - 0 = 0$

(b) $|r'(t)| = \sqrt{(4\cos t)^2 + 3^2 + (-4\sin t)^2} = 5$

$\vec{T}(t) = \frac{1}{5}\langle 4\cos t, 3, -4\sin t\rangle$

$\vec{T}\,'(t) = \frac{1}{5}\langle -4\sin t, 0, -4\cos t\rangle \Rightarrow |\vec{T}\,'(t)| = \frac{1}{5}\sqrt{4^2(\sin^2 t + \cos^2 t)} = \frac{4}{5}$

$\vec{N}(t) = \frac{\frac{1}{5}\langle -4\sin t, 0, -4\cos t\rangle}{4/5} = \langle -\sin t, 0, \cos t\rangle$

$t=\pi$: $\vec{N}(\pi) = \langle 0, 0, 1\rangle$, $\vec{T}(\pi) = \frac{1}{5}\langle 4, 3, 0\rangle$

$T \times N = \frac{1}{5}(3\vec{i} - 4\vec{j})$ so $\langle 3, -4, 0\rangle$ is normal vector for osc. plane

$\Rightarrow 3x - 4(y - 3\pi) + 0\cdot(z+4) = 0$

Please turn page

3. *(5 points)* Find the curvature of $\vec{r}(t) = \langle t, t^2, 2t+1 \rangle$ as a function of t.

$\vec{r}'(t) = \langle 1, 2t, 2 \rangle$

$|\vec{r}'(t)| = \sqrt{1^2 + 4t^2 + 2^2} = \sqrt{5+4t^2}$

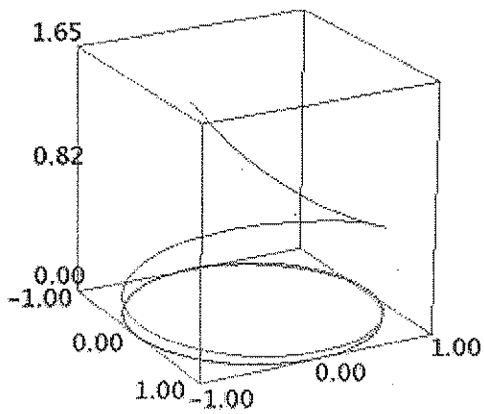
$\vec{r}''(t) = \langle 0, 2, 0 \rangle$

$$K = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|^3} = \frac{|\langle -4, 0, 2 \rangle|}{\sqrt{5+4t^2}^{\,3}} = \frac{\sqrt{20}}{(5+4t)^{3/2}}$$

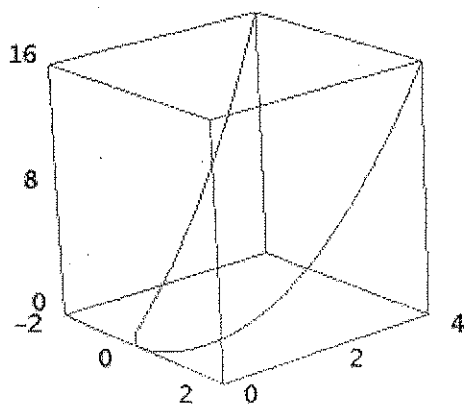
4. *(4 points)* Match the following equations with the graphs below:

$\vec{r}(t) = \langle t, \cos(5t), \sin(5t) \rangle$	III	$\vec{r}(t) = \langle \cos(2t), \sin(2t), e^{-t} \rangle$	I
$\vec{r}(t) = \langle t, t^2, t^4 \rangle$	II	$\vec{r}(t) = \langle t\cos(10t), t\sin(10t), t \rangle$	IV

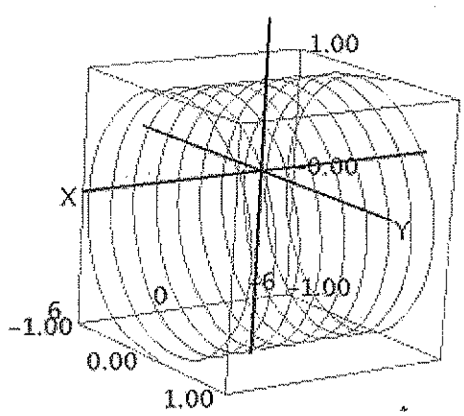
I



II



III



IV