Your UID: ____________________

Midterm 2 (2110 Multivariabls Calculus, Fall 2009)

November 19, 2009

70 min. No symbolic calculators allowed (TI-89 and similar)!

(TI-86 or lower are allowed.)

Show all work.

Your name: ____________________

Score (out of 65)	Letter grade

Score: ______

1. *(10 points)*

   (a) Compute the partial derivatives of $f(x, y) = e^x y^2$.

   (b) Compute an equation for the tangent plane of the graph of $z = f(x, y)$ at the point $(x, y) = (0, 3)$.

   (c) Given an approximation of $f(0.01, 0.98)$ using the linear approximation of $f(x, y)$ at the point $(0, 1)$.

Score: ______

2. *(6 points)* The function $F(x, y)$ is given by $F(x, y) = u^2 e^{u+v}$ and $u(x, y) = x + y$, $v(x, y) = x - y$. Compute the partial derivative $\frac{\partial F}{\partial y}$ and write it **in terms of** x **and** y.

3. *(6 points)* Compute the following iterated integral by changing the order of integration. *Hint:* First make a sketch of the region over which a corresponding double integral would integrate.

$$\int_0^2 \int_{2x}^4 4e^{-y^2}\, dy\, dx =$$

Score: ______

4. *(10 points)*

(a) Find all critical points of

$$f(x,y) = xy + \frac{1}{x} + \frac{1}{y}$$

(b) Determine whether your critical point(s) is(are) a local minimum, a local maximum, or a saddle point.

Score: ______

5. *(8 points)* Let D be the area outside of the unit circle $x^2 + y^2 = 1$, inside the circle $x^2 + y^2 = 2$, above the line $y = x$, and to the right of $x = 0$. Compute the double integral

$$\iint_D xy\, dA =$$

6. *(5 points)* Let T be the body outside the cylinder $x^2 + y^2 = 9$, inside the cylinder $x^2 + y^2 = 16$, above the plane $z = 0$ and below the cone $z^2 = x^2 + y^2$. Set up an iterated integral that computes the volume of T.

   You don't need to compute the integral. *Hint:* You should choose whether to use cylindrical or spherical coordinates.

Score: ______

7. *(5 points)* Let T be the body bounded by $z = 0$, $x = 0$, $y = 0$ and by $z = 1 - x^2 - y$. Rewrite the following integral as an iterated integral: **You don't need to compute the integral.**

$$\iiint_T x \ dV =$$

8. *(5 points)* Let D be the region in the plane bounded by $x = 1$, by $x = 2$, by $xy = 1$ and by $xy = 2$. Use the transformation $x = u, y = \frac{v}{u}$ to compute the integral

$$\iint_D =$$

Hint: First compute the Jacobian of the transformation. Then translate the equation for every boundary into an equation between u and v.

9. *(10 points)* Let $\vec{F}(x,y)$ be the vector field $\vec{F}(x,y) = \langle xy, \frac{1}{2}x^2 + 3y^2 \rangle$.

(a) Show that $\vec{F}(x,y)$ is a conservative vector field.

(b) Find a function $f(x,y)$ such that $\vec{F}(x,y) = \nabla f(x,y)$.

(c) Let C be the curve given by

$$\vec{r}(t) = \langle \cos(\pi t), \sin(\tfrac{\pi}{2}t) \rangle \quad \text{for } 0 \leq t \leq 1.$$

Compute the line integral

$$\int_C \vec{F} \bullet d\vec{r} =$$

Score: ______