

ERRATA

Updated March 3, 2003

Page 17, line 1: The integral should be

$$\int_0^t g'_n(|X_s - X'_s|) \operatorname{sgn}(X_s - X'_s) [b(X_s) - b(X'_s)] ds$$

Page 21, line 2: and $X_t \rightarrow \infty$ a.s.Page 24, lines 5-15: where $|e_s|$ is bounded, say by c_2 (cf. PTA, theorem I.5.11). Let $\tau = \inf\{t > 0 : |X_{B_t}^1| > N\}$. Since

$$|X_{B_{n+1}}^1 - X_{B_n}^1| \geq |\widetilde{W}_{n+1} - \widetilde{W}_n| - c_2,$$

then $|X_{B_{n+1}}^1| > N$ on the set $\{|X_{B_n}^1| < N, |\widetilde{W}_{n+1} - \widetilde{W}_n| > c_2 + 2N\}$. There exists $c_3 \in (0, 1)$ such that $\mathbb{P}(|\widetilde{W}_{n+1} - \widetilde{W}_n| > c_2 + 2N) > c_3$. By the independent increment property of Brownian motion,

$$\begin{aligned} \mathbb{P}(\tau > n + 1) &= \mathbb{P}(\tau > n + 1, \tau > n) = \mathbb{E}[\mathbb{P}(\tau > n + 1 \mid \mathcal{F}_n); \tau > n] \\ &\leq (1 - c_3)\mathbb{P}(\tau > n). \end{aligned}$$

By induction, $\mathbb{P}(\tau > n) \leq (1 - c_3)^n$; hence $\tau < \infty$ a.s. Since $d\langle M \rangle_t/dt$ is

Page 25, lines 11-12:

Theorem 8.4. *Let W_t be a d -dimensional Brownian motion and let X_t be a d -dimensional process such that*

$$X_t^i = x_0^i + \int_0^t \sum_{j=1}^d H_{ij}(s) dW_s^j + \int_0^t B_i(s) ds, \quad i = 1, \dots, d,$$

where H_{ij} and B_i are predictable and bounded. For each s and ω let $K(s)$ be the matrix that is the inverse of $H(s)$. Suppose there exists M such that for all s, i , and j , $|H_{ij}(s)|$, $|K_{ij}(s)|$, and $|B_i(s)|$ are bounded by M a.s. Let $\varepsilon > 0$, $t_0 > 0$. There exists c_1 depending only on M such that

Page 44, line 7: $\Lambda(x) > 0$ Page 44, Display (1.2): $\sum_{i=1}^d y_i^2$ Page 45, line -3: $e^{-\lambda s} \lambda u(X_s) ds$

Page 77, line -2: > 0 .

Page 160, lines 15–21: Replace by the following.

If $x, y \in Q$,

$$u(x) - u(y) = \int_0^{|y-x|} \partial_r u(y + rv) dr, \quad v = (x - y)/|y - x|.$$

Integrating with respect to y ,

$$|Q|[u(x) - u_Q] = \int_Q \int_0^{|y-x|} \partial_r u(x + rv) dr dy.$$

Set $V(z)$ equal to $|\nabla u(z)|$ if $z \in Q$ and 0 otherwise. Then

$$\begin{aligned} |u(x) - u_Q| &\leq \frac{1}{|Q|} \int_{|y-x| \leq 2\sqrt{a}} \int_0^\infty V(x + rv) dr dy \\ &\leq c_2 \int_Q |y - x|^{1-d} V(y) dy. \end{aligned}$$

Now apply this inequality together with Theorem IV.5.1 of [PTA] where we set $p = 2$ and we set $K(x, y) = |y - x|^{1-d}$ if $x, y \in Q$ and 0 otherwise. We obtain

$$\int_Q |u(x) - u_Q|^2 dx \leq c_2^2 \int_Q \left[\int_Q K(x, y) V(y) dy \right]^2 dx \leq c_3 \int_Q |\nabla u(x)|^2 dx.$$

Page 188. Replace the proof of Theorem 8.4 by the following.

Proof. As in the proof of Theorem 7.5, we may assume without loss of generality that $d \geq 3$. As in the proof of Theorem I.8.5, it suffices to consider the case where ψ is differentiable. By Proposition 6.7 there exists c_2 and c_3 such that if $|x - x_0| \leq c_2 r^{1/2}$ and $|x - y| \leq c_2 r^{1/2}$, then $p_{B(x_0, r^{1/2})}(x, y) \geq c_3 r^{-d/2}$. Choose n large so that if $r = t/n$, then $r^{1/2} \leq \varepsilon/8$ and $r \|\psi'\|_\infty \leq (c_2/2)r^{1/2}$. Let $y_i = \psi(ir)$. Let $c_4 = c_2/4$.

If $x \in B(y_i, c_4 r^{1/2})$ and $y \in B(y_{i+1}, c_4 r^{1/2})$, then

$$|x - y| \leq 2c_4 r^{1/2} + |y_i - y_{i+1}| \leq 2c_4 r^{1/2} + r \|\psi'\|_\infty \leq c_2 r^{1/2}.$$

Taking $x_0 = y_i$, we see $p_{B(y_i, r^{1/2})}(x, y) \geq c_3 r^{-d/2}$. It follows that

$$\mathbb{P}^x(X_r \in B(y_{i+1}, r^{1/2}), \sup_{s \leq r} |X_s - X_0| \leq \varepsilon/4) \geq c_2 r^{-d/2} |B(y_i, r^{1/2})| \geq c_5.$$

Note

$$\begin{aligned} &\mathbb{P}^{\psi(0)}(\sup_{s \leq t} |X_s - \psi(s)| < \varepsilon) \\ &\geq \mathbb{P}^{\psi(0)}(X_{ir} \in B(y_i, c_4 r^{1/2}), \sup_{s \leq r} |X_s - X_{ir}| \leq \varepsilon/4, i = 0, \dots, n), \end{aligned}$$

and applying the Markov property n times, this is greater than $c_5^n > 0$. \square

Page 202, lines -10, -9: Replace by the following:

Since

$$\begin{aligned} 0 &= I - I = V(W + \varepsilon H_j)V^{-1}(W + \varepsilon H_j) - V(W)V^{-1}(W) \\ &= (V(W + \varepsilon H_j) - V(W))V^{-1}(W) \\ &\quad + V(W + \varepsilon H_j)(V^{-1}(W + \varepsilon H_j) - V^{-1}(W)), \end{aligned}$$

then

$$V^{-1}(W + \varepsilon H_j) - V^{-1}(W) = -V^{-1}(W + \varepsilon H_j)(V(W + \varepsilon H_j) - V(W))V^{-1}(W).$$

Dividing both sides by ε and letting $\varepsilon \rightarrow 0$,