1. Fill in the following table by listing the values for sine and cosine at the given t values.

	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos(t)$	1	0	-1	0	1
$\sin(t)$	0	1	0	-1	0

2. Fill in the following table.

	Definition	Domain	Range	Period	Max/Min
$\cos(t)$	N/A	$(-\infty, \infty)$	$[-1, 1]$	2π	$1, -1$
$\sin(t)$	N/A	$(-\infty, \infty)$	$[-1, 1]$	2π	$1, -1$
$\tan(t)$	$\frac{\sin t}{\cos t}$	$t \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$	$(-\infty, \infty)$	π	$\infty, -\infty$
$\cot(t)$	$\frac{\cos t}{\sin t}$	$t \neq 0, \pm\pi, \pm 2\pi, \ldots$	$(-\infty, \infty)$	π	$\infty, -\infty$
$\sec(t)$	$\frac{1}{\cos t}$	$t \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots$	$(-\infty, -1] \cup [1, \infty)$	2π	$\infty, -\infty$
$\csc(t)$	$\frac{1}{\sin t}$	$t \neq 0, \pm\pi, \pm 2\pi, \ldots$	$(-\infty, -1] \cup [1, \infty)$	2π	$\infty, -\infty$

(See the graphs of these functions on the last page of this review sheet.)

3. (a) Fill in the following table by listing the values for sine and cosine at the given t values.

	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\cos(t)$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\sin(t)$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$

(b) List all angles that can be re-written as a sum or difference of $\pi/6$, $\pi/4$, and $\pi/3$.

(i) $\frac{\pi}{6} + \frac{\pi}{4} = \frac{5\pi}{12}$

(ii) $\frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2}$

(iii) $\frac{\pi}{4} + \frac{\pi}{3} = \frac{7\pi}{12}$

(iv) $\frac{\pi}{6} - \frac{\pi}{4} = -\frac{\pi}{12}$

(v) $\frac{\pi}{6} - \frac{\pi}{3} = -\frac{\pi}{6}$

(vi) $\frac{\pi}{4} - \frac{\pi}{3} = -\frac{\pi}{12}$

(c) Determine the value of the following trigonometric functions using a **sum or difference formula**.

i. $\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{\pi}{4}+\frac{\pi}{3}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{3}\right)+\sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\cdot\frac{1}{2}+\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}+\sqrt{6}}{4}$

ii. $\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{\pi}{6}+\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right)-\sin\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}-\frac{\sqrt{1}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$

iii. $\sin\left(-\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{6}-\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right)-\sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) = \frac{1}{2}\cdot\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{2}-\sqrt{6}}{4}$

iv. $\cos\left(-\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{6}-\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{4}\right)+\sin\left(\frac{\pi}{6}\right)\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}+\frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$

(d) Determine the value of the following trigonometric functions using a **half-angle formula**.

i. $\sin\left(\frac{7\pi}{12}\right) = \sin\left(\frac{1}{2}\cdot\frac{7\pi}{6}\right) = \sqrt{\frac{1-\cos\left(\frac{7\pi}{6}\right)}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2+\sqrt{3}}}{2}$ *(second quadrant, $y>0$)*

ii. $\cos\left(\frac{5\pi}{12}\right) = \cos\left(\frac{1}{2}\cdot\frac{5\pi}{6}\right) = \sqrt{\frac{1+\cos\left(\frac{5\pi}{6}\right)}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2-\sqrt{3}}}{2}$ *(first quadrant, $x>0$)*

iii. $\sin\left(-\frac{\pi}{12}\right) = \sin\left(\frac{1}{2}\cdot\frac{-\pi}{6}\right) = -\sqrt{\frac{1-\cos\left(\frac{-\pi}{6}\right)}{2}} = -\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = -\frac{\sqrt{2-\sqrt{3}}}{2}$

(fourth quadrant, $y<0$)

iv. $\cos\left(-\frac{\pi}{12}\right) = \cos\left(\frac{1}{2}\cdot\frac{-\pi}{6}\right) = \sqrt{\frac{1+\cos\left(\frac{-\pi}{6}\right)}{2}} = \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}} = \frac{\sqrt{2+\sqrt{3}}}{2}$ *(fourth quadrant, $x>0$)*

Editor's note: The values in (c) and (d) are the *same!*

(e) Use the answers from (c) to calculate the following values.

i. $\cos\left(\frac{7\pi}{12}\right) = -\sqrt{1-\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)^2} = -\sqrt{\frac{8-4\sqrt{3}}{16}} = -\frac{\sqrt{2-\sqrt{3}}}{2} = \frac{\sqrt{2}-\sqrt{6}}{4}$

(second quadrant, $x<0$)

ii. $\sin\left(\frac{5\pi}{12}\right) = \sqrt{1-\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)^2} = \sqrt{\frac{8+4\sqrt{3}}{16}} = \frac{\sqrt{2+\sqrt{3}}}{2} = \frac{\sqrt{2}+\sqrt{6}}{4}$

(first quadrant, $y>0$)

Editor's note: If you do the same calculations with the answers in (d), you will get the same answers.

(f) Use the answers from (c) and (e) to calculate the following values.

i. $\tan\left(\frac{7\pi}{12}\right) = \frac{\sqrt{2}+\sqrt{6}}{\sqrt{2}-\sqrt{6}} = -\frac{\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}$

ii. $\cot\left(\frac{5\pi}{12}\right) = \frac{\sqrt{6}-\sqrt{2}}{\sqrt{2}+\sqrt{6}} = \frac{\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}$

iii. $\sec\left(-\frac{\pi}{12}\right) = \frac{4}{\sqrt{6}+\sqrt{2}} = \frac{2}{\sqrt{2+\sqrt{3}}}$

iv. $\csc\left(-\frac{\pi}{12}\right) = \frac{4}{\sqrt{2}-\sqrt{6}} = -\frac{2}{\sqrt{2-\sqrt{3}}}$

4. Let $f(x) = 2\sin(3x-\pi)+1$.

(a) Determine the amplitude, period, and range of f

Amplitude $=2$, Period $=\frac{2\pi}{3}$, Range $=[-1,3]$

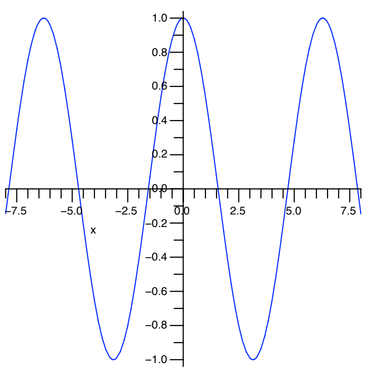
(b) Write f as a cosine function; that is, find A, B, C, and D so that $f(x) = A\cos(Bx+C)+D$.

Use the fact that $\sin(t) = \cos\left(t-\frac{\pi}{2}\right)$ and let $t = 3x-\pi$.

$$f(x) = 2\sin(3x-\pi)+1 = 2\cos\left((3x-\pi)-\frac{\pi}{2}\right)+1 = 2\cos\left(3x-\frac{3\pi}{2}\right)+1$$

5. Use your knowledge of the graphs of sin, cos and tan to sketch the following. In addition, state the period and amplitude of each.

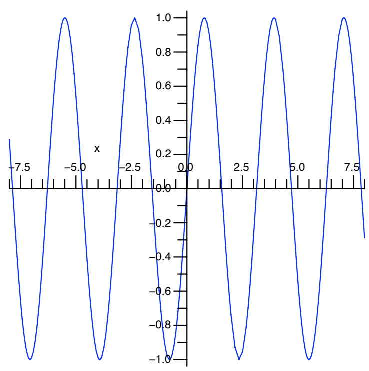
(a) $y = \cos(x - 2\pi)$



Period: 2π

Amplitude: 1

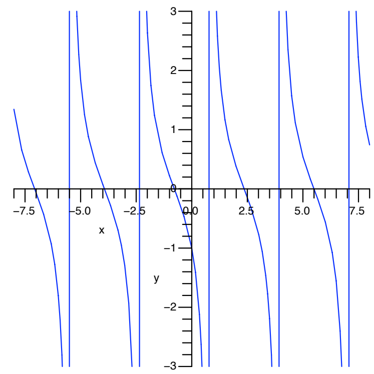
(b) $y = -\sin(2x - \pi)$



Period: π

Amplitude: 1

(c) $y = -\tan(x + \pi/4)$



Period: π

Amplitude: ∞

6. Suppose a circle has radius 9.

(a) Find the length of the arc on the circle, which is intercepted by a central angle of 75°.

$$= 9 \cdot \left(75 \cdot \frac{2\pi}{360}\right) = \frac{75\pi}{20}$$

(b) What is the area of the circular sector (or wedge) formed by the central angle of 75°?

$$= \frac{1}{2} \cdot 9^2 \cdot \left(75 \cdot \frac{2\pi}{360}\right) = \frac{675\pi}{40}$$

7. (a) Find the length of an arc formed by an angle of $7\pi/6$ radians in a circle of radius 3

$$= 3 \cdot \frac{7\pi}{6} = \frac{7\pi}{2}$$

(b) Find the area of a sector (or wedge) formed by an angle of 130° in a circle of radius 3

$$= \frac{1}{2} \cdot 3^2 \cdot \left(130 \cdot \frac{2\pi}{360}\right) = \frac{65\pi}{20}$$

8. Given that $\sin t = 1/3$ and that $\pi/2 \le t \le \pi$, find the value of the following.
   (a) $\sin(-t) = -\frac{1}{3}$
   (b) $\csc(-t) = -3$
   (c) $\cos(t) = -\sqrt{\frac{8}{9}} = -\frac{2\sqrt{2}}{3}$ *(second quadrant, $x < 0$)*
   (d) $\sec(t) = -\frac{3}{2\sqrt{2}}$

9. What is the domain of the function $f(x) = \tan(2x)$?

$$\left\{x : \cos(2x) \neq 0\right\} = \left\{x : 2x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \dots\right\} = \left\{x : x \neq \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, \pm\frac{5\pi}{4}, \dots\right\}$$

10. Evaluate the six basic trig functions at $t = 17\pi/6$.
Observe that $\frac{17\pi}{6} = \frac{12\pi}{6} + \frac{5\pi}{6} = 2\pi + \frac{5\pi}{6}$.

(a) $\cos\left(\frac{17\pi}{6}\right) = \cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$ (d) $\sec\left(\frac{17\pi}{6}\right) = -\frac{2}{\sqrt{3}}$

(b) $\sin\left(\frac{17\pi}{6}\right) = \sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$ (e) $\csc\left(\frac{17\pi}{6}\right) = 2$

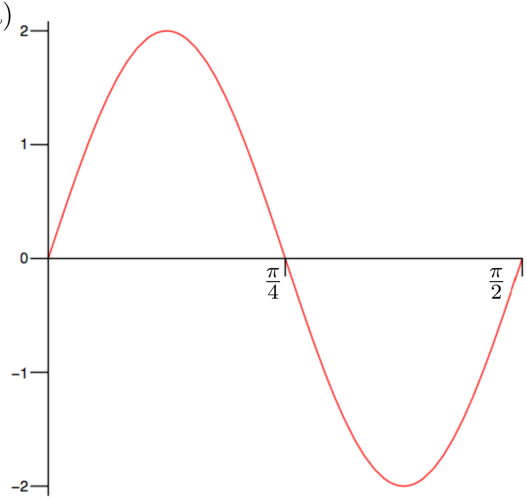
(c) $\tan\left(\frac{17\pi}{6}\right) = \frac{1/2}{-\sqrt{3}/2} = -\frac{1}{\sqrt{3}}$ (f) $\cot\left(\frac{17\pi}{6}\right) = -\sqrt{3}$

11. Find all values of t in the interval $[0, 2\pi]$ that satisfy the equation $2\sin(t) + \sqrt{3} = 0$.
We must find all t so that $\sin(t) = -\frac{\sqrt{3}}{2}$. The t in the interval $[0, 2\pi]$ are: $t = \frac{4\pi}{3}$, $t = \frac{5\pi}{3}$.
($y = \sin(t) < 0$, so we must be in the third or fourth quadrant.)

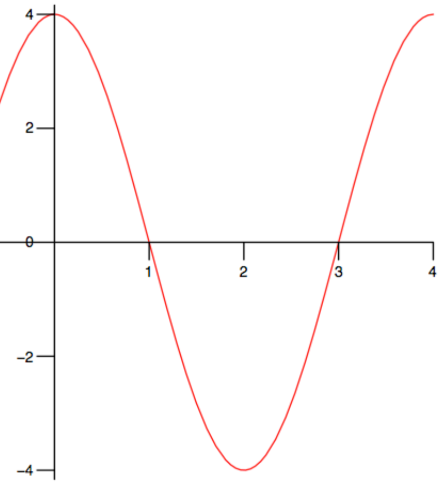
12. Given that $\sec\theta = \frac{13}{5}$ for $0 \le \theta \le \frac{\pi}{2}$, evaluate the remaining five trig functions at θ.
   (a) $\cos\theta = \frac{5}{13}$
   (b) $\sin\theta = \sqrt{1 - \frac{5^2}{13^2}} = \frac{12}{13}$ *(first quadrant, $y > 0$)*
   (c) $\tan\theta = \frac{12/13}{5/13} = \frac{12}{5}$
   (d) $\csc\theta = \frac{13}{12}$
   (e) $\cot\theta = \frac{5}{12}$

13. Find a sine function AND a cosine function whose graph matches the given curve.

(a)



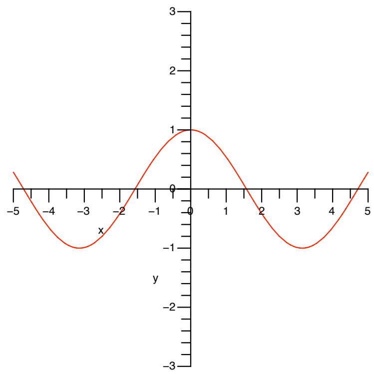
(b)



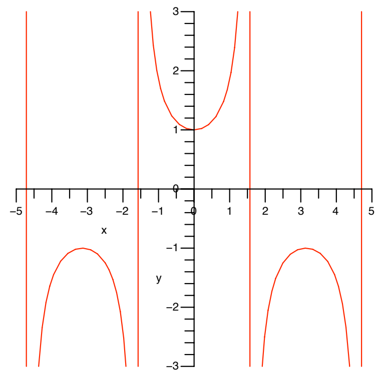
(a) $f(x) = 2\sin(4x) = 2\cos\left(4x - \frac{\pi}{2}\right)$

(b) $g(x) = 4\cos\left(\frac{\pi}{2}x\right) = 4\sin\left(\frac{\pi}{2}x + \frac{\pi}{2}\right)$

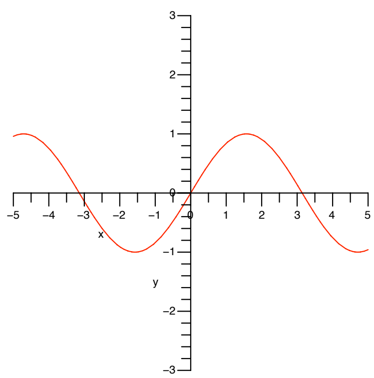
$y = \cos x$



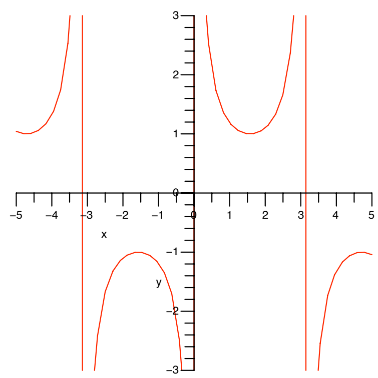
$y = \sec x$



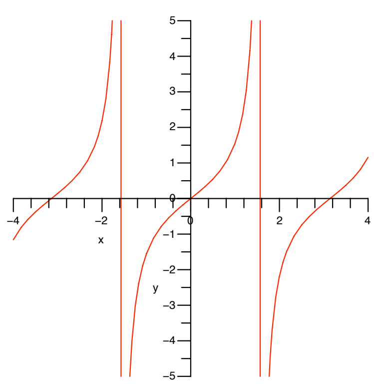
$y = \sin x$



$y = \csc x$



$y = \tan x$



$y = \cot x$