Final Exam Review Sheet
Math 109

1. Give, in slope-intercept form, the equation of the line that:
   (a) Passes through the two points $(3, 5)$ and $(-2, 4)$.
   (b) Passes through the point $(2, 3)$ and is parallel to the line $y = 2x - 5$.
   (c) Passes through the point $(1, 4)$ and is parallel to the line $2x + 3y = 6$.
   (d) Passes through the point $(-3, -1)$ and is perpendicular to the line $y = 3x + 5$.
   (e) Passes through the point $(4, 7)$ and is perpendicular to the line $y = 5$.

2. Find the values of x that satisfy the given inequality. Write your answers in interval notation.
   (a) $2x - 5 \geq 3$
   (b) $3x + 4 < 5$
   (c) $|1 - 2x| \leq 5$
   (d) $|1 - 2x| > 5$
   (e) $x^2 - 5x + 6 \geq 0$

3. (a) Find the distance between the points $(3, 4)$ and $(1, 2)$.
   (b) Find the equation of the circle with center at $(3, 4)$ that passes through the point $(1, 2)$. Write the equation in standard form.
   (c) Is the point $(4, 2)$ on the circle in Part (b)?

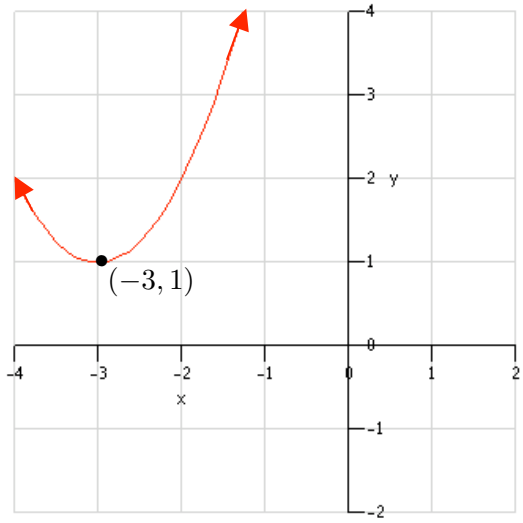
4. Let $f(x) = \sqrt{x^2 - 3x - 4}$. Calculate the following. Simplify, if possible.
   (a) Find $f(4)$.
   (b) Find $f(-2)$.
   (c) Find $f(-x)$.
   (d) Find $f(x + 2)$.
   (e) Find the domain of the function f.

5. Consider the function $f(x) = x^2 - 9$.
   (a) Find the y-intercept(s) of f. Write each intercept as an ordered pair (x, y). If there are no y-intercepts, write "NONE."
   (b) Find the x-intercept(s) of f. Write each intercept as an ordered pair (x, y). If there are no x-intercepts, write "NONE."

6. Consider the function $g(x) = 3x^2 + 2x - 1$.
   (a) Find the y-intercept(s) of f. Write each intercept as an ordered pair (x, y). If there are no y-intercepts, write "NONE."
   (b) Find the x-intercept(s) of f. Write each intercept as an ordered pair (x, y). If there are no x-intercepts, write "NONE."

7. Consider the function $f(x) = x^2 + 4$.
   (a) Find the y-intercept(s) of f. Write each intercept as an ordered pair (x, y). If there are no y-intercepts, write "NONE."
   (b) Find the x-intercept(s) of f. Write each intercept as an ordered pair (x, y). If there are no x-intercepts, write "NONE."

8. Below is the graph of the function g.



   (a) State the domain and range of the function g.
   (b) Give a one or two sentence explanation of why g is a function.
   (c) Does g have any of the three symmetries discussed in class? If so, which?
   (d) Sketch the graph of the equation $y = -g(x - 3) + 2$ on the same coordinate plane as $y = g(x)$.

9. Let $f(x) = 3 - 5x^2$ and $g(x) = \sqrt{2 - x}$. Calculate the following.
   (a) $(f - g)(-2) =$
   (b) $(f \cdot g)(1) =$
   (c) $\left(\frac{f}{g}\right)(x) =$
   (d) $(f \circ g)(x) =$
   (e) What is the domain of the function $f \circ g$?
   (f) $(f \circ g)(3) =$

10. Let $h(x) = \dfrac{x^3}{3x^2 - 4}$ and $k(x) = 3x^2 - 4$. Calculate the following.
    (a) What is the domain of the function h?
    (b) $(k + h)(2) =$
    (c) $(h \cdot k)(x) =$
    (d) What is the domain of the function $h \cdot k$?
    (e) $(h \cdot k)\left(\dfrac{2}{\sqrt{3}}\right) =$
    (f) $(h \circ k)\left(\dfrac{2}{\sqrt{3}}\right) =$
    (g) If $f(x) = \sqrt{x^2 + 4}$, then what is the domain of the function $h \circ f$?

11. Let
$$P(x) = x^3 - 2x^2 - x + 2.$$
    (a) Explain why $x - 2$ is a factor of $P(x)$.
    (b) $P(x)$ has three rational zeros. Factor $P(x)$ completely.

12. Let
$$Q(x) = 3x^8 - 12x^6 + 23x^5 + 15x^3 - 12x - 10.$$
    (a) What is the degree of $Q(x)$?
    (b) What is the greatest number of zeros that $Q(x)$ can have? Why?
    (c) Use the Rational Zero (Root) Test to list all possible rational zeros of $Q(x)$.
    (d) What is the leading coefficient of $Q(x)$?
    (e) What is the constant term of $Q(x)$?

13. Let
$$P(x) = 2x^3 - x^2 - 3x - 1.$$
    (a) Use the Rational Zero (Root) Test to list all possible rational zeros of $P(x)$.
    (b) Find all the zeros of $P(x)$.
    (c) Factor $P(x)$ completely.

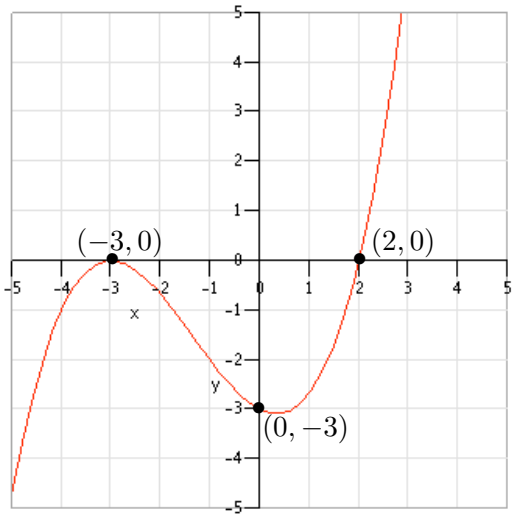
14. Let $f(x) = \sqrt{2 - x}$.
    (a) Sketch the graph of $y = f(x)$.
    (b) State the domain of $f(x)$.
    (c) State the range of $f(x)$.
    (d) The inverse of $f(x)$ exists. Give a one to two sentence explanation of why this is true.
    (e) Determine an expression for $f^{-1}(x)$.
    (f) Sketch the graph of $y = f^{-1}(x)$.

15. Let
$$R(x) = \frac{x^2 - 3x + 2}{x^2 + 3x + 2}.$$
   (a) State the domain of $R(x)$.
   (b) Find the x-intercept(s) of $R(x)$. Write each intercept as an ordered pair, or write "none" if there are no x-intercepts.
   (c) Find the y-intercept(s) of $R(x)$. Write each intercept as an ordered pair, or write "none" if there are no y-intercepts.
   (d) Find the vertical asymptote(s) of $R(x)$. Give the equation of each asymptote, or write "none" if there are no vertical asymptotes.
   (e) Find the horizontal asymptote(s) of $R(x)$. Give the equation of each asymptote, or write "none" if there are no horizontal asymptotes.

16. Let $R(x) = \dfrac{x^2}{3x^4 + 5x^2 + 1}$. Find the horizontal asymptote(s) of $R(x)$.

17. Let $R(x) = \dfrac{x^2}{3x^2 + 5x + 1}$. Find the horizontal asymptote(s) of $R(x)$.

18. The graph of $y = g(x)$ is below.
   (a) Sketch the graph of $y = g(x - 2) + 2$ on the same axes.



   (b) If $g(x)$ is a polynomial, what is the smallest possible degree of $g(x)$?
   (c) If $g(x)$ is a polynomial, what are the factors of $g(x)$?
   (d) If $g(x)$ is a polynomial, what might be its equation?

19. Let $t = 20°$. Give the **exact** value for each of the following questions.
   (a) Find the measurement of t in radians. Simplify your result.
   (b) In a circle of radius 5, what is the length of an arc with angle of t radians (20°)?

20. Let $t = 120°$. Give the **exact** value for each of the following questions.
    (a) Find the measurement of t in radians. Simplify your result.
    (b) In a circle of radius 4, what is the area of a circular sector with angle of t radians (120°)?

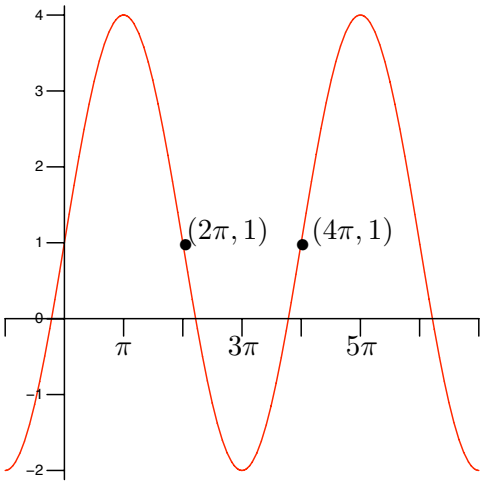
21. Suppose $\sin\theta = \frac{2}{3}$ where $0 < \theta < \pi/2$. Calculate the following.
    (a) $\cos\theta =$
    (b) $\tan\theta =$
    (c) $\sec\theta =$
    (d) $\csc\theta =$
    (e) $\cot\theta =$

22. Suppose $\sin\theta = \frac{1}{4}$ where $\pi/2 < \theta < \pi$. Calculate the following.
    (a) $\cos\theta =$
    (b) $\sin(-\theta) =$
    (c) $\cos(-\theta) =$
    (d) $\sin(\theta + 2\pi) =$
    (e) $\sin(\theta + 6\pi) =$
    (f) $\sin(\theta + \pi) =$
    (g) $\sin(\theta + \frac{\pi}{2}) =$

23. Determine the exact value of the following using a trigonometric identity.
    (a) $\sin\left(\frac{7\pi}{12}\right) =$
    (b) $\cos\left(\frac{\pi}{12}\right) =$
    (c) $\sin\left(\frac{5\pi}{12}\right) =$
    (d) $\cos\left(\frac{5\pi}{12}\right) =$
    (e) $\sin\left(-\frac{7\pi}{12}\right) =$
    (f) $\cos\left(\frac{13\pi}{12}\right) =$
    (g) $\cos\left(\frac{25\pi}{12}\right) =$

24. Consider the function
$$f(t) = 3\sin\left(5t + \frac{\pi}{4}\right).$$
    (a) Find the amplitude and period of f.
    (b) Write f as a cosine function.

25. Consider the function
$$g(t) = 2\cos(\pi t) + 3.$$
    (a) Find the amplitude and period of g.
    (b) Write g as a sine function.



26. Above is the graph of the $y = f(x)$. Give all answers as **exact** values.
    (a) State the domain and range of f.
    (b) State the period and amplitude of f.
    (c) Write an equation for f.

27. Use a sum or difference formula to show the following:
    (a) $\cos(t + 2\pi) = \cos(t)$
    (b) $\sin(t + \pi/2) = \cos(t)$

28. (a) Find two different values of t so that $\sin(t) = \frac{\sqrt{3}}{2}$.
    (b) Find the value of $\arcsin(\frac{\sqrt{3}}{2})$.

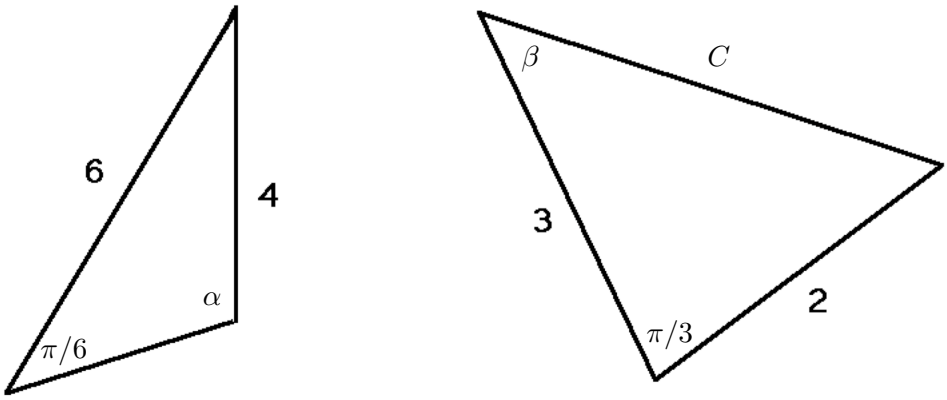
29. Find all values of x in the interval $[0, 2\pi]$ which make the following equation true:

$$2\cos x - \sqrt{2} = 0$$

30. Calculate the following:
    (a) $\arccos(-1/2) =$
    (b) $\arcsin(-1/2) =$
    (c) $\arctan(\sqrt{3}) =$
    (d) $\arcsin(3) =$

31. Calculate the following:
    (a) $\sin(\arccos(3/4)) =$
    (b) $\tan(\arcsin(2/5)) =$
    (c) $\cos(\arctan(3)) =$
    (d) $\sin(\arctan(x)) =$

32. Use the figure below to answer the following questions. In the figure, α and β are the angle measures.



Calculate the following:

(a) $\sin\alpha =$

(b) $\cos\alpha =$

(c) $\sin(2\alpha) =$

(d) $\sin(\alpha/2) =$

(e) $C =$

(f) $\sin\beta =$

(g) $\cos\beta =$

(h) $\sin(\alpha+\beta) =$

(i) $\cos(\alpha+\beta) =$

33. Let $f(x) = 3^{x-2} + 1$.

(a) Sketch the graph of $y = f(x)$. Label any 3 points and indicate any asymptotes.

(b) What is the domain of f?

(c) What is the range of f?

(d) Describe how you would sketch the graph of $y = 2f(x)$.

(e) Does the inverse of f exist? If so, calculate the formula for f^{-1}.

(f) Write f in terms of the natural base, e.

34. Let $g(x) = -\ln(x+1)$.
    (a) Sketch the graph of $y = g(x)$. Label any 3 points and indicate any asymptotes.
    (b) What is the domain of g?
    (c) What is the range of g?
    (d) Describe how you would sketch the graph of $2g(x)$.
    (e) Does the inverse of g exist? If so, calculate the formula for g^{-1}.

35. Rewrite each expression as a single logarithm.
    (a) $3\ln x - \dfrac{1}{2}\ln(x+2)$
    (b) $\ln 2 + 4\ln(x+7)$
    (c) $\log_3 x + 2\log_3(x-1) + 3\log_3(x+1)$

36. Solve each equation for x.
    (a) $\ln x + \ln(x-1) = \ln 2$
    (b) $\log_4(10-x) = 2$
    (c) $2\ln x = \ln 2 + \ln(x+4)$