Final Exam Review Sheet Solutions
Math 109

1. (a) $y = \frac{1}{5}x + \frac{22}{5}$
   (b) $y = 2x - 1$
   (c) $y = -\frac{2}{3}x + \frac{14}{3}$
   (d) $y = -\frac{1}{3}x - 2$
   (e) $x = 4$

2. (a) $[4, \infty)$
   (b) $\left(-\infty, \frac{1}{3}\right)$
   (c) $[-2, 3]$
   (d) $(-\infty, -2) \cup (3, \infty)$
   (e) $(-\infty, 2] \cup [3, \infty)$

3. (a) $\sqrt{(3-1)^2 + (4-2)^2} = \sqrt{8} = 2\sqrt{2}$
   (b) $(x-3)^2 + (y-4)^2 = 8$
   (c) No : $\quad (4-3)^2 + (2-4)^2 = 5 \neq 8$

4. (a) 0
   (b) $\sqrt{6}$
   (c) $\sqrt{x^2 + 3x - 4}$
   (d) $\sqrt{x^2 + x - 6}$
   (e) $(-\infty, -1] \cup [4, \infty)$

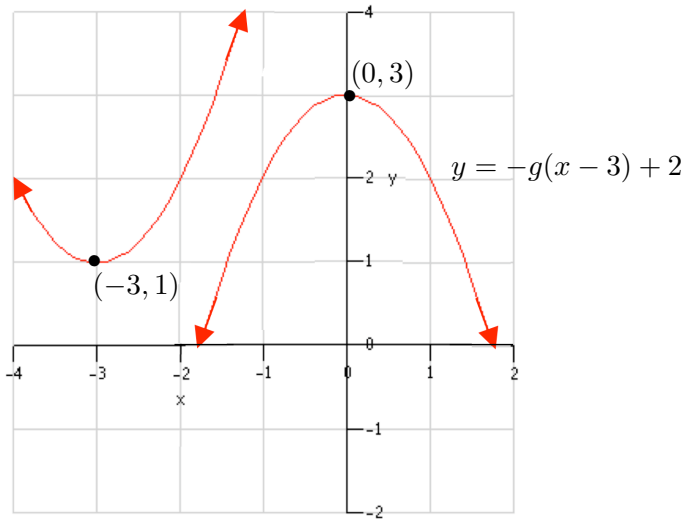
5. (a) $(0, -9)$
   (b) $(3, 0), \quad (-3, 0)$

6. (a) $(0, -1)$
   (b) $\left(\frac{1}{3}, 0\right), \quad (-1, 0)$

7. (a) $(0, 4)$
   (b) NONE

8. (a) Domain : $(-\infty, \infty)$ Range : $[1, \infty)$
   (b) It passes the *vertical* line test.
   (c) No, it does not posses x-axis, y-axis, nor origin symmetry .
   (d) See the graph below.

9. (a) $(-17) - (2) = -19$



(b) $(-2)(1) = -2$

(c) $\dfrac{3-5x^2}{\sqrt{2-x}}$

(d) $3-5\left(\sqrt{2-x}\right)^2 = -7+5x$

(e) $\{x : x \le 2\} = (-\infty, 2]$

(f) Undefined: 3 is not in the domain of $f \circ g$.

10. (a) $\left\{x : x \neq \pm\dfrac{2}{\sqrt{3}}\right\} = \left(-\infty, -\dfrac{2}{\sqrt{3}}\right) \cup \left(-\dfrac{2}{\sqrt{3}}, \dfrac{2}{\sqrt{3}}\right) \cup \left(\dfrac{2}{\sqrt{3}}, \infty\right)$

(b) $8 + \dfrac{8}{8} = 9$

(c) $\left(\dfrac{x^3}{3x^2-4}\right) \cdot \left(3x^2-4\right) = x^3$

(d) $\left\{x : x \neq \pm\dfrac{2}{\sqrt{3}}\right\} = \left(-\infty, -\dfrac{2}{\sqrt{3}}\right) \cup \left(-\dfrac{2}{\sqrt{3}}, \dfrac{2}{\sqrt{3}}\right) \cup \left(\dfrac{2}{\sqrt{3}}, \infty\right)$

(e) Undefined

(f) $h\left(k(2/\sqrt{3})\right) = h(0) = 0$

(g) $3\left(\sqrt{x^2+4}\right)^2 - 4 = 3x^2 + 8 > 0$, so the domain is $(-\infty, \infty)$

11. (a) $P(2) = 0$

(b) $P(x) = (x-2)(x-1)(x+1)$

12. (a) 8

(b) Eight. The number of zeros cannot be more than the degree.

(c) $\pm 1, \pm 2, \pm 5, \pm 10, \pm\dfrac{1}{3}, \pm\dfrac{2}{3}, \pm\dfrac{5}{3}, \pm\dfrac{10}{3}$

(d) 3

(e) -10

13. (a) $\pm 1, \pm\dfrac{1}{2}$

    (b) $-\dfrac{1}{2}, \quad \dfrac{1+\sqrt{5}}{2}, \quad \dfrac{1-\sqrt{5}}{2},$

    (c) $P(x) = 2\left(x+\dfrac{1}{2}\right)\left(x-\dfrac{1+\sqrt{5}}{2}\right)\left(x-\dfrac{1-\sqrt{5}}{2}\right)$

14. (a) See graph below

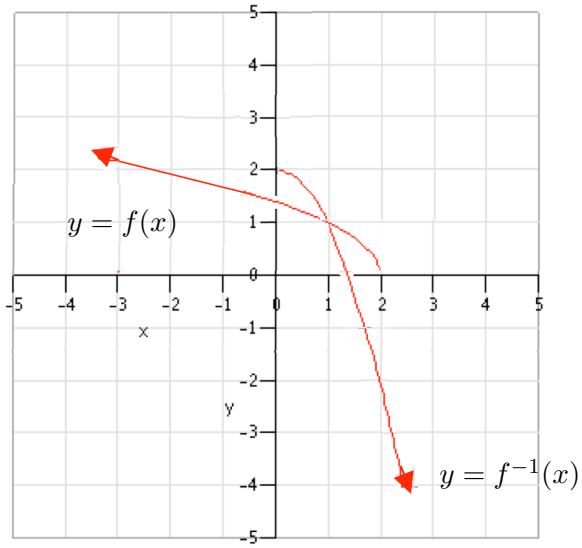
    (b) $(-\infty, 2]$

    (c) $[0, \infty)$

    (d) f passes the horizontal line test, so is one-to-one.

    (e) $f^{-1}(x) = 2 - x^2$

    (f)



15. (a) $\{x : x \neq -1, -2\} = (-\infty, -2) \cup (-2, -1) \cup (-1, \infty)$

    (b) $(1, 0), \quad (2, 0)$

    (c) $(0, 1)$

    (d) $x = -1, \quad x = -2$

    (e) $y = 1$

16. $y = 0$

17. $y = \dfrac{1}{3}$

18. (a) Translate right 2 and up 2.

(b) 3

(c) $x+3, \quad x-2$

(d) $g(x) = \dfrac{1}{6}(x+3)^2(x-2)$

19. (a) $\pi/9$

(b) $5\pi/9$

20. (a) $2\pi/3$

(b) $16\pi/3$

21. (a) $\dfrac{\sqrt{5}}{3}$

(b) $\dfrac{2}{\sqrt{5}}$

(c) $\dfrac{3}{\sqrt{5}}$

(d) $\dfrac{3}{2}$

(e) $\dfrac{\sqrt{5}}{2}$

22. (a) $-\dfrac{\sqrt{15}}{4}$

(b) $-\sin\theta = -\dfrac{1}{4}$

(c) $\cos\theta = -\dfrac{\sqrt{15}}{4}$

(d) $\sin\theta = \dfrac{1}{4}$

(e) $\sin\theta = \dfrac{1}{4}$

(f) $-\sin\theta = -\dfrac{1}{4}$

(g) $\cos\theta = -\dfrac{\sqrt{15}}{4}$

23. (a) $\dfrac{\sqrt{2}+\sqrt{6}}{4} = \dfrac{\sqrt{2+\sqrt{3}}}{2}$

(b) $\dfrac{\sqrt{6}+\sqrt{2}}{4} = \dfrac{\sqrt{2+\sqrt{3}}}{2}$

(c) $\dfrac{\sqrt{2+\sqrt{3}}}{2} = \dfrac{\sqrt{2}+\sqrt{6}}{4}$

(d) $\dfrac{\sqrt{6}-\sqrt{2}}{4} = \dfrac{\sqrt{2-\sqrt{3}}}{2}$

(e) $-\sin\left(\dfrac{7\pi}{12}\right) = -\dfrac{\sqrt{2}+\sqrt{6}}{4} = -\dfrac{\sqrt{2+\sqrt{3}}}{2}$

(f) $-\cos\left(\dfrac{\pi}{12}\right) = -\dfrac{\sqrt{6}+\sqrt{2}}{4} = -\dfrac{\sqrt{2+\sqrt{3}}}{2}$

(g) $\cos\left(\dfrac{\pi}{12}\right) = \dfrac{\sqrt{6}+\sqrt{2}}{4} = \dfrac{\sqrt{2+\sqrt{3}}}{2}$

24. (a) $A = 3, \quad P = 2\pi/5$

(b) $f(t) = 3\cos\left(5t - \dfrac{\pi}{4}\right)$

25. (a) $A = 2, \quad P = 2$

(b) $g(t) = 2\sin\left(\pi t + \dfrac{\pi}{2}\right) + 3$

26. (a) $D : (-\infty, \infty), \quad R : [-2, 4]$

(b) $A = 3, \quad P = 4\pi$

(c) $f(x) = 3\sin\left(\dfrac{x}{2}\right) + 1$

27. (a) $\cos(t + 2\pi) = \cos(t)\cos(2\pi) - \sin(t)\sin(2\pi) = \cos(t)(1) - \sin(t)(0) = \cos(t)$

(b) $\sin(t + \pi/2) = \sin(t)\cos(\pi/2) + \sin(2\pi)\cos(t) = \sin(t)(0) + (1)\cos(t) = \cos(t)$

28. (a) $t = \dfrac{\pi}{3}, \ \dfrac{2\pi}{3}, \ -\dfrac{4\pi}{3}, \ \ldots$ (there are many such t)

(b) $t = \dfrac{\pi}{3}$ (there is only one such t)

29. $x = \dfrac{\pi}{4}, \ \dfrac{7\pi}{4}$

30. (a) $\dfrac{2\pi}{3}$

(b) $-\dfrac{\pi}{6}$

(c) $\dfrac{\pi}{3}$

(d) Undefined: 3 is not in the domain of arcsine.

31. (a) $\dfrac{\sqrt{7}}{4}$

(b) $\dfrac{2}{\sqrt{21}}$

(c) $\dfrac{1}{\sqrt{10}}$

(d) $\dfrac{x}{\sqrt{1+x^2}}$

32. (a) $\dfrac{3}{4}$
    (b) $\dfrac{\sqrt{7}}{4}$
    (c) $\dfrac{3\sqrt{7}}{8}$
    (d) $\sqrt{\dfrac{4-\sqrt{7}}{8}}$
    (e) $\sqrt{7}$
    (f) $\sqrt{\dfrac{3}{7}}$
    (g) $\dfrac{2}{\sqrt{7}}$
    (h) $\dfrac{3}{2\sqrt{7}} + \dfrac{\sqrt{3}}{4}$
    (i) $\dfrac{1}{2} - \dfrac{3\sqrt{3}}{4\sqrt{7}}$

33. (a)
    (b) $(-\infty, \infty)$
    (c) $[1, \infty)$
    (d) For every x, multiply the corresponding y by 2.
    (e) $f^{-1}(x) = \log_3(x-1) + 2 = \dfrac{\ln(x-1)}{\ln 3} + 2$
    (f) $f(x) = e^{(x-2)\ln 3} + 1$

34. (a)
    (b) $(-1, \infty)$
    (c) $(-\infty, \infty)$
    (d) For every x, multiply the corresponding y by 2.
    (e) $g^{-1}(x) = e^{-x} - 1$

35. (a) $\ln\left(\dfrac{x^3}{\sqrt{x+2}}\right)$
    (b) $\ln\left[2(x+7)^4\right]$
    (c) $\log_3\left[x(x-1)^2(x+1)^3\right]$

36. (a) $x = 2, \;\; x = -1$ (extraneous solution)
    (b) $x = -6$
    (c) $x = 4, \;\; x = -2$ (extraneous solution)