ELLIPTIC CURVES WITH COMPLEX MULTIPLICATION

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Abstract. This is a summary of the table that can be found in [Sil94], page 483.

The following table (Table 1) contains the minimal Weierstrass equation of elliptic curves $E/\mathbb{Q}$ with complex multiplication by the full ring of integers of the quadratic imaginary field of discriminant D_K. The j-invariant j_E, the discriminant Δ_E and the conductor N_E of the elliptic curve E are also given.

In Table 2 one can find the equation of a p-twist of the elliptic curves with complex multiplication. However, the equation of the twist is not necessarily minimal.

List of Tables

D_K	Minimal Weierstrass Equation	j_E	Δ_E	N_E
-3	$y^2+y=x^3$	0	3^3	3^3
-4	$y^2=x^3+x$	$2^6 3^3$	2^6	2^6
-7	$y^2+xy=x^3-x^2-2x-1$	$-3^3 5^3$	7^3	7^2
-8	$y^2=x^3+4x^2+2x$	$2^6 5^3$	2^9	2^8
-11	$y^2+y=x^3-x^2-7x+10$	-2^{15}	11^3	11^2
-19	$y^2+y=x^3-38x+90$	$-2^{15}3^3$	19^3	19^2
-43	$y^2+y=x^3-860x+9707$	$-2^{18}3^3 5^3$	43^3	43^2
-67	$y^2+y=x^3-7370x+243528$	$-2^{15}3^3 5^3 11^3$	67^3	67^2
-163	$y^2+y=x^3-2174420x+1234136692$	$-2^{18}3^3 5^3 23^3 29^3$	163^3	163^2

Table 1. Elliptic Curves with Complex Multiplication

D_K	Minimal Weierstrass Equation	Equation of p-Twist
-3	$y^2+y=x^3$	$y^2=x^3+16p^3$
-4	$y^2=x^3+x$	$y^2=x^3+p^2x$
-7	$y^2+xy=x^3-x^2-2x-1$	$y^2=x^3-3px^2-32p^2x-64p^3$
-8	$y^2=x^3+4x^2+2x$	$y^2=x^3+4px^2+2p^2x$
-11	$y^2+y=x^3-x^2-7x+10$	$y^2=x^3-4px^2-112p^2x+656p^3$
-19	$y^2+y=x^3-38x+90$	$y^2=x^3-608p^2x+5776p^3$
-43	$y^2+y=x^3-860x+9707$	$y^2=x^3-13760p^2x+621264p^3$
-67	$y^2+y=x^3-7370x+243528$	$y^2=x^3-117920p^2x+15585808p^3$
-163	$y^2+y=x^3-2174420x+1234136692$	$y^2=x^3-34790720p^2x+78984748304p^3$

TABLE 2. Twists of Elliptic Curves with Complex Multiplication

REFERENCES

[Sil94] J. H. Silverman, *Advanced Topics in the Arithmetic of Elliptic Curves*. Springer-Verlag, New York, 1994.

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