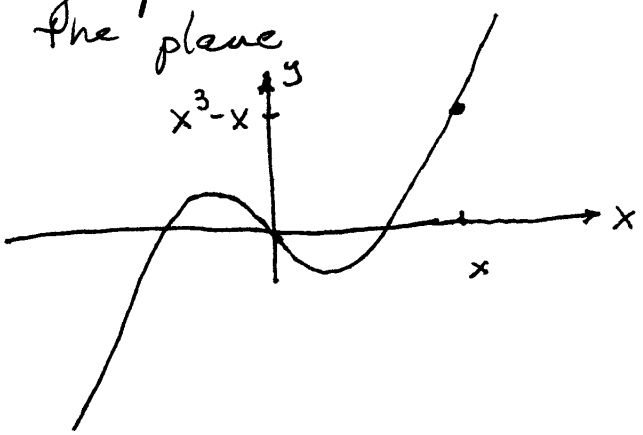
The Graph of a Function.

We start with our intuition

On the reals, we define $y = f(x) = x^3 - x$

Its graph is a curve (nice collection of points) in the plane



the points on the curve are of the form

$$(x, y) = (x, x^3 - x)$$

(x, y) are thought of as cartesian coordinates.

We can also graph data - then the points might be

(sample #, sample value) $= (n, f(n))$

$f(n)$ could even be multidimensional (temperature, barometric pressure, wind speed, etc.)

Def'n: Given two set X & Y, the (Cartesian) product of X & Y, denoted $X \times Y$ is the set of pairs

$$\{(x, y) \mid x \in X, y \in Y\}$$

If $X = Y$, we often write X^2 instead of $X \times X$.

The graph of the function as shown above, lives in $\mathbb{R}^2$.

which is the plane.

$\mathbb{R}^3$ is 3-space $= \mathbb{R} \times \mathbb{R} \times \mathbb{R} = \{(x,y,z) \mid x, y, z \in \mathbb{R}\}$.

Defn - Suppose $f: X \to Y$ is some function. Then the graph of f is the subset of $X \times Y$ given by

$$\{(x, f(x)) \mid x \in X\}.$$

* When is a subset of $X \times Y$ the graph of a function?