Tubular surfaces of varying radii

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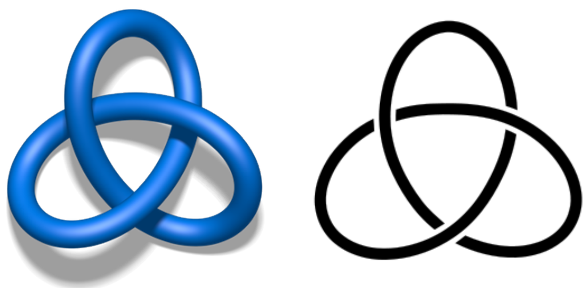
Tubular surfaces (the boundaries of tubular neighborhoods) have both theoretical and practical applications. For instance, tubular surfaces can be used to represent knot types as diagrams, as Figure 1 shows[1]. More fundamentally, a non-self-intersecting tubular surface plays an important role in the preservation of knot structure during approximations. Precisely it specifies a tubular neighborhood within which the approximants obtain the same knot structure as the given curve, under certain conditions. This can be applied to computer visualization and simulations.



(a) A tubular surface (b) A diagram

Figure 1: Trefoil Knots

Earlier published work [1] only considered a tubular surface of constant radius. However I found that the algorithms based on non-self-intersecting tubular surfaces of varying radii are more efficient.

Theorem 1 *For a curve in $\mathbb{R}^3$, there exists a non-self-intersecting tubular surface of varying radii such that it obtains the maximal radii that yield no self-intersections. This tubular surface is the optimal solution among all non-self-intersecting tubular surfaces.*

Another future research direction is studying how to obtain efficiency by switching a tubular surface of constant radius to that of varying radii.

References

[1] T. Maekawa, N. M. Patrikalakis, T. Sakkalis, and G. Yu. Analysis and applications of pipe surfaces. *CAGD*, 15(5):437–458, 1998.

[1] Imagecredits:http://en.wikipedia.org/wiki/Trefoil_knot.