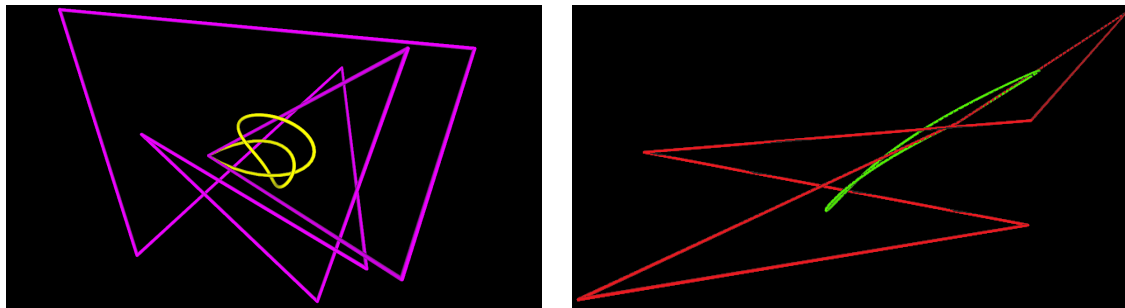


Computational Topology Counterexamples with 3D Visualization of Bézier Curves

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I worked with a recently developed knot visualization tool to investigate the topological features including knottedness and self-intersections. I found a “knotted” Bézier curve determined by an “unknotted” control polygon as Figure 1(a) shows to further demonstrate the topological differences. It was motivated by a recently constructed dual example of an “unknotted” Bézier curve with a “knotted” control polygon [1]. I also gave rigorous mathematical proofs of these topological properties. Proving knottedness and unknottedness is usually difficult. But this work suggests a method of showing knottedness and unknottedness under certain circumstances.



(a) Knot vs. Unknot

(b) Self-intersecting curve vs. Equilateral polygon

Figure 1: Knots examples

A Bézier curve and the control polygon are not necessarily homeomorphic as Section ?? shows. But an empirical study of dozens of cases under the visualization tool showed that Bézier curves were “simple” when their simple control polygons were “equilateral”. This empirical evidence led a conjecture that the additional equilateral hypothesis on a simple control polygon ensures the homeomorphism. I then developed a numerical algorithm to search counterexamples and it indeed succeeded, as Figure 1(b) shows a self-intersecting Bézier curve determined by a equilateral simple control polygon. Similar examples can be easily generated by the algorithm.

References

- [1] J. Bisceglia, T. J. Peters, J. A. Roulier, and C. H. Sequin. Unknots with highly knotted control polygons. *Computer Aided Geometric Design*, 28(3):212–214, 2011.