Rates of Convergence in the Central Limit Theorem for Markov Chains

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University of Connecticut, 2012

ABSTRACT

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Doctor of Philosophy Dissertation

Rates of Convergence in the Central Limit Theorem for Markov Chains

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NOTATION

A) **Spaces**

- $\mathscr{L}^2(\mathbb{R}^d)$, all square integrable functions.
- $\mathscr{D}(\mathbb{R}^d)$, all cadlag functions.
- $\mathscr{C}(\mathbb{R}^d, \mathbb{R})$, all continuous functions from $\mathbb{R}^d$ to $\mathbb{R}$.
- $\mathscr{C}_0(\mathbb{R}^d, \mathbb{R})$, those functions in $\mathscr{C}(\mathbb{R}^d, \mathbb{R})$ that vanish at infinity.
- $\mathscr{C}_{00}(\mathbb{R}^d, \mathbb{R})$, those functions in $\mathscr{C}(\mathbb{R}^d, \mathbb{R})$ that have compact support.

B) **General Operators**

- $\mathcal{L}, \mathcal{M}, \hat{\mathcal{L}}, \hat{\mathcal{M}}$: generators of a semigroup
- $\mathcal{P}_t, \mathcal{Q}_t, \hat{\mathcal{P}}_t, \hat{\mathcal{Q}}_t$: the corresponding semigroups

C) **Specific Operators**

- $Lf(x) = \nabla \cdot A(x) \nabla f(x)$
- $\tilde{L}f(x) = \nabla \cdot \tilde{A}(x) \nabla f(x)$
- $L^h f(x) = \frac{1}{h^2} \sum\limits_{z \in \mathbb{Z}^d} (f(x+hz) - f(x))\, C_{\frac{x}{h}, \frac{x}{h}+z}$
- $\tilde{L}^h f(x) = \frac{1}{h^2} \sum\limits_{z \in \mathbb{Z}^d} (f(x+hz) - f(x))\, \tilde{C}^{\varepsilon}_{\frac{x}{h}, \frac{x}{h}+z}$
- Semigroups: $P_t = e^{tL}$, $\tilde{P}_t = e^{t\tilde{L}}$, $P^h_t = e^{tL^h}$, $\tilde{P}^h_t = e^{t\tilde{L}^h}$

D) **Markov Processes**

- $X^{(h)}_t$ is the process on $h\mathbb{Z}^d$ with generator L^h.
- $\tilde{X}^{(h)}_t$ is the process on $h\mathbb{Z}^d$ with generator $\tilde{L}^h$.
- X_t is the process on $\mathbb{R}^d$ with generator L.
- $\tilde{X}_t$ is the process on $\mathbb{R}^d$ with generator $\tilde{L}$.

E) **Dirichlet Forms**
In general, $\mathscr{E}_{\mathcal{M}}(f,g) = -\langle \mathcal{M} f | g \rangle$. In $\mathscr{L}^2(\mathbb{R}^d, \mu)$, the inner product is defined as

$$\langle f | g \rangle = \int_{\mathbb{R}^d} f(x) g(x) dx$$

while in $\mathscr{L}^2(h\mathbb{Z}^d, \mu^h)$ it is defined as

$$\langle f | g \rangle = h^d \sum_{x \in h\mathbb{Z}^d} f(x) g(x).$$

Note the rescaling factor h^d. Therefore, the various Dirichlet forms are

- $\mathscr{E}(f,g) = -\int [\nabla \cdot A \nabla f](x)\, g(x)\, dx$
- $\tilde{\mathscr{E}}(f,g) = -\int [\nabla \cdot \tilde{A} \nabla f](x)\, g(x)\, dx$
- $\mathscr{E}^h(f,g) = \frac{1}{2} h^{d-2} \sum_{x,z \in \mathbb{Z}^d} (f(x+hz) - f(x))\, (g(x+hz) - g(x))\, C_{\frac{x}{h}, \frac{x}{h}+z}$
- $\tilde{\mathscr{E}}^{h,\varepsilon}(f,g) = \frac{1}{2} h^{d-2} \sum_{x,z \in \mathbb{Z}^d} (f(x+hz) - f(x))\, (g(x+hz) - g(x))\, \tilde{C}^{\varepsilon}_{\frac{x}{h}, \frac{x}{h}+z}$

F) **Our Standard Mollifier**
For each $\varepsilon > 0$, we define a standard mollifier:

$$\eta_\varepsilon(y) = \frac{1}{\varepsilon^d} \eta\left(\frac{y}{\varepsilon}\right)$$

where η is defined as follows:

$$\eta(y) = \begin{cases} c_d \exp\left(\frac{1}{\|y\|^2 - 1}\right) & \text{if } \|y\| < 1 \\ 0 & \text{otherwise} \end{cases}$$

with c_d chosen such that $\|\eta\|_1 = 1$.

G) **Miscellaneous**

- c is a positive constant for which the actual value does not matter and can change from line to line. Sometimes an index will be added for clarity.

Introduction

Setting the Problem

0.1 The First Section

0.2 The Second Section

0.3 The Third Section

Chapter 1

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1.1 The First Section

1.2 The Second Section

1.3 The Third Section

1.4 The Fourth Section

Chapter 2

The Second Chapter Title Goes Here

2.1 The First Section

2.2 The Second Section

2.3 The Third Section

2.4 The Fourth Section

Bibliography

[1] David Applebaum, *Lévy Processes and Stochastic Calculus*, Cambridge Studies in Advanced Mathematics, ISBN 0-521-83263-2, Cambridge University Press, 2004

[2] Richard F. Bass, *Probabilistic Techniques in Analysis*, Probability and its Applications, ISBN 0-387-94387-0, Springer Verlag, 1995

[3] Richard F. Bass, *Diffusions and Elliptic Operators*, Probability and its Applications, ISBN 0-387-98315-5, Springer Verlag, 1998

[4] Richard F. Bass and Takashi Kumagai, *Symmetric Markov Chains on $\mathbb{Z}^d$ with Unbounded Range*, Transactions of the American Mathematical Society, Vol. 30, Nr 4, 2041-2075 (2008), AMS, 2008

[5] Zhen-Qing Chen, Zhongmin Qian, Yaozhong Hu and Weian Zheng, *Stability and Approximations of Symmetric Diffusion Semigroups and Kernels*, Journal of Functional Analysis, 152, 225-280 (1998), Academic Press, 1998

[6] Rama Cont and Peter Tankov, *Financial modelling with jump processes*, Financial Mathematics Series, ISBN 1-5848-8413-4, Chapman & Hall/CRC, 2004

[7] A. De Masi, P.A. Ferrari, S. Goldstein, W.D. Wick, *An Invariance Principle for Reversible Markov Processes*, Journal of Statistical Physics, Vol. 55, 787-855 (1989), Springer Verlag, 1989

[8] Joseph L. Doob, *Classical Potential Theory and Its Probabilistic Counterpart*, Classics in Mathematics ISBN 3-540-41206-9, Springer Verlag, 1984

[9] Richard Durrett, *Probability Theory and Examples*, Wadsworth & BrooksCole StatisticsProbability Series, ISBN 0-534-13206-5, BrooksCole Publishing Company, 1991

[10] Richard Durrett, *Stochastic Calculus*, Probability and Stochastics Series, ISBN 0-8493-8071-5, CRC Press, 1996

[11] Lawrence C. Evans, *Partial Differential Equations*, Graduate Studies in Mathematics, ISBN 0-821-80772-2, American Mathematical Society, 1998

[12] Warren P. Johnson, *The Curious History of Faà di Bruno's Formula*, American Mathematical Monthly, Vol. 109, 217-234, (March 2003), MAA, 2003

[13] Ioannis Karatzas and Steven Schreve, *Brownian Motion and Stochastic Calculus* (2nd Ed.), ISBN 0-387-97655-8, Springer Verlag, 1988, 1991

[14] Jean-Pierre Lepeltier and Bernard Marchal, *Problème des martingales et équations différentielles stochastiques associées à un oprateur inégro-différentiel*, Annales de l' Institut Henri Poincaré (B) Vol. 12, 43-103, (1976)

[15] Zhi-Ming Ma and Michael Röckner, *Introduction to the Theory of (Non-symmetric) Dirichlet Forms*, ISBN 3-540-55848-9, Springer Verlag, 1992

[16] Bernt Øksendal, *Stochastic Differential Equations* (6th Ed.), ISBN 3-540-04758-1, Springer Verlag, 1985, 2003

[17] L.C.G. Rogers and D. Williams, *Diffusions, Markov Processes and Martingales, Vol. II : Itô Calculus* (2nd Ed.), ISBN 0-521-77593-0, Cambridge University Press, 2000

[18] Walter Rudin, *Functional Analysis*, ISBN 0-070-54236-8, McGraw-Hill Science and Engineering, 1991

[19] A.V. Skorokhod, *Studies in the Theory of Random Processes*, ISBN 9-780-48664240-6, Reading, Mass., Addison-Wesley Pub. Co., 1965

[20] D.W. Stroock and W. Zheng, *Markov Chain Approximations to Symmetric Diffusions*, Annales de l'Institut Henri Poincaré (B), Vol. 33, 619-649, (1997)