Problem 1. Let $X$ be the set of all points $(x, y) \in \mathbb{R}^2$ such that $y = 1$ or $y = -1$. Let $M$ be the quotient of $X$ by the equivalence relation generated by $(x, -1) \sim (x, 1)$ for all $x \neq 0$. Show that $M$ is not Hausdorff.

Problem 2. Suppose $f : X \rightarrow Y$ is a continuous bijection, $X$ is compact, and $Y$ is Hausdorff. Prove that $f$ is a homeomorphism.

Problem 3. Show that if a path-connected, locally path-connected space $X$ has $\pi_1(X)$ finite, then every map $X \rightarrow \mathbb{T}^2$ is nullhomotopic.

Problem 4. Let $A$ be a subset of a topological space $X$. Suppose that $r : X \rightarrow A$ is a retraction of $X$ onto $A$, i.e. $r$ is a continuous map such that the restriction of $r$ to $A$ is the identity map of $A$.

(1) Show that if $X$ is Hausdorff, then $A$ is a closed subset.

(2) Let $a \in A$. Show that $r_* : \pi_1(X, a) \rightarrow \pi_1(A, a)$ is surjective.

Problem 5. Let $S^n$ be an $n$-dimensional sphere in $\mathbb{R}^{n+1}$ centered at the origin. Suppose $f, g : S^n \rightarrow S^n$ are continuous maps such that $f(x) \neq -g(x)$ for any $x \in S^n$. Prove that $f$ and $g$ are homotopic.

Problem 6. Let $k \geq 1$ be an integer. Compute the fundamental groups of the following spaces.

(1) The sphere $S^2$ with $k$ points removed.

(2) The torus $\mathbb{T}^2$ with $k$ points removed.