1. Let \( X, Y \) be topological spaces, \( Y \) Hausdorff, and let \( A \subset X \) be a non-empty set.

   (a) Suppose that \( f : A \to Y \) is continuous, where \( A \) is equipped with the subspace topology. Prove that if there exists a continuous extension of \( f \) to \( \overline{A} \), it is unique.

   (b) Assume that \( A \) is connected in the subspace topology. Prove that \( \overline{A} \) is connected in the subspace topology.

2. Let \( S \) denote the standard topology on \( \mathbb{R} \) and let \( T \) be the topology on \( \mathbb{R} \) generated by the intervals \( [a, b) \), where \( a \in \mathbb{Q} \) and \( b \in \mathbb{R} \).

   (a) Suppose that \( f : (\mathbb{R}, T) \to (\mathbb{R}, S) \) is a function. Show that \( f \) is continuous if and only if the function \( g : (\mathbb{R}, S) \to (\mathbb{R}, S) \) given by \( g(x) = f(x) \) is right-continuous at all rational points and continuous at all irrational points.

   (b) Is \( (\mathbb{R}, T) \) metrizable?

3. Suppose that \( X \) is a topological space homeomorphic to an open subset of a compact Hausdorff space. Prove that \( X \) is locally compact (=every point has a neighborhood contained in a compact set).

4. The real projective plane \( P^2 \) is the topological space of lines in \( \mathbb{R}^3 \) passing through the origin. One construction of \( P^2 \) is as a quotient space of the unit sphere \( S^2 = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\} \) with the subspace topology, obtained by identifying antipodal points.

   (a) Prove that \( P^2 \) is compact Hausdorff and that every point in \( P^2 \) has a neighborhood homeomorphic to the open unit ball in \( \mathbb{R}^2 \).

   (b) Prove that the quotient map is a covering map.

   (c) Find a path in \( S^2 \) whose image under the quotient map generates the fundamental group for \( P^2 \) based at the image of \((1, 0, 0)\) under the quotient map.

5. Show that \( \mathbb{R}^3 \) is not homeomorphic to \( \mathbb{R}^2 \).

6. Let \( X \) be the subspace of \( \mathbb{R}^3 \) equal to the union of the unit sphere with the three line segments \( \{(0, 0, z) : |z| \leq 1\} \cup \{(0, y, 0) : |y| \leq 1\} \cup \{(x, 0, 0) : |x| \leq 1\} \). Compute the fundamental group of \( X \) based at \((1, 0, 0)\).