Problem 1. Let $X$ be a topological space and let $A$ be a subset of $X$. Either prove the following statement, or give a counter-example.

1. If $A$ is connected, then the closure $\overline{A}$ is connected.

2. If $A$ is connected, then its interior $\text{Int}(A)$ is connected.

Problem 2. Define the equivalence relation on $\mathbb{R}$ such that $x \sim y$ if $x - y$ is rational. Let $\mathbb{R}/\sim$ be the quotient space with the quotient topology. Show that $\mathbb{R}/\sim$ is not Hausdorff.

Problem 3. Let $X,Y$ be topological spaces. Assume that $Y$ is Hausdorff. Let $f,g : X \to Y$ be continuous functions. Suppose that there exists a dense subset $D$ of $X$ such that $f(x) = g(x)$ for all $x \in D$. Prove that $f(x) = g(x)$ for all $x \in X$.

Problem 4. A topological space $X$ is said to be contractible if the identity map $\text{Id}_X : X \to X$ is null-homotopic, i.e. homotopic to a constant map.

1. Show that any convex subset of $\mathbb{R}^n$ is contractible.

2. Let $Y$ be a topological space. Show that if $X$ is contractible, then any map $f : X \to Y$ is null-homotopic.

Problem 5. Let $E,X$ be topological spaces. Assume that $E$ is connected. Let $q : E \to X$ be a covering map with $q^{-1}(x)$ finite and non-empty for all $x \in X$. Show that $E$ is compact if and only if $X$ is compact.

Problem 6. Let $n \geq 3$. Suppose that $M$ is a connected $n$-dimensional manifold, and $p \in M$. Show that the inclusion $M \setminus \{p\} \hookrightarrow M$ induces an isomorphism between their fundamental groups $\pi_1(M \setminus \{p\}) \cong \pi_1(M)$.