# Topology Prelim, August 2014

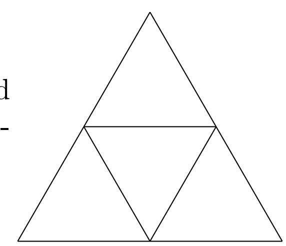
1. Let $\mathcal{D}$ denote the topology on $\mathbb{Z}$ generated by sets of the form $\{2n-1, 2n, 2n+1\}$, $n \in \mathbb{Z}$. Prove:

   (a) Given two distinct elements in $\mathbb{Z}$, then there exists a $\mathcal{D}$-neighborhood of one which does not contain the other, yet $(\mathbb{Z}, \mathcal{D})$ is not Hausdorff.

   (b) $(\mathbb{Z}, \mathcal{D})$ is connected.

2. Let $A$ and $B$ be subsets of a topological space $X$ so that $A \bigcup B$ and $A \bigcap B$ are connected. Prove that if $A$ and $B$ are closed, then both $A$ and $B$ are connected.

3. Let $X \subset \mathbb{R}^2$ be the subspace in the figure on the right, and let $A$ denote the top vertex of the external triangle. Use Van-Kampen's Theorem to find $\pi_1(X, A)$.



4. Let $\mathbb{D} = \{z \in \mathbb{C} : |z| \leq 1\}$, and let $\sim$ be the equivalence relation on $\mathbb{D}$, given by

$$z_1 \sim z_2 \Leftrightarrow \begin{cases} z_1 = e^{i\theta_1}, z_2 = e^{i\theta_2},\ \theta_1, \theta_2 \in [0, 2\pi) \text{ and } 3(\theta_2 - \theta_1) \equiv 0 \mod 2\pi \\ z_1 = z_2 \text{ otherwise} \end{cases}$$

   Compute $\pi_1(\mathbb{D}/\sim, [1])$, where $[1]$ is the equivalence class of $1 \in \mathbb{C}$ with respect to $\sim$. (Recall Euler's formula: $e^{i\theta} = \cos\theta + i \sin\theta$).

5. (a) Define: $(\tilde{X}, p)$ is a covering space of a topological space $X$.

   (b) Find a simply connected covering space for the subspace of $\mathbb{R}^3$ given by the union of a sphere and its diameter.