1. Let $X$ and $Y$ be connected spaces. If $A$ is a proper subset of $X$ and $B$ is a proper subset of $Y$ then $X \times Y - A \times B$ is connected. ( $A$ is a proper subset of $X$ if $A \subset X$ and $A \neq X$.)

2. Let $X$ be a first countable space.

   (a) For any set $A \subset X$ and any point $p \in X$, show that $p \in A$ if and only if there is a sequence $\{p_n\}_{n=1}^{\infty}$ in $A$ such that $\{p_n\}$ converges to $p$.

   (b) Show that for any space $Y$, a map $f : X \rightarrow Y$ is continuous if and only if $f$ takes convergent sequences in $X$ to convergent sequences in $Y$.

3. (a) If $X$ is a locally connected space then prove that the components of $X$ are open subsets of $X$.

   (b) Let $p : X \rightarrow Y$ be a quotient map. Show that if $X$ is locally connected, then $Y$ is locally connected. (Hint: If $C$ is a component of an open set $U$ of $Y$, show that $p^{-1}(C)$ is a union of components of $p^{-1}(U)$.)

4. Let $X$ be a Hausdorff space. Suppose that $\{A_\alpha \mid \alpha \in \mathcal{A}\}$ is a collection of compact, connected subsets of $X$ simply ordered by inclusion (that is, for each $\alpha, \beta \in \mathcal{A}$ we have either $A_\alpha \subset A_\beta$ or $A_\beta \subset A_\alpha$). Prove that $\cap_{\alpha \in \mathcal{A}} A_\alpha$ is nonempty and connected.

5. A continuous map $f : X \rightarrow X$ is called a retraction of $X$ onto $A = f(X)$ if $f \circ f = f$. The image $A$ of $f$ is called a retract of $X$.

   (a) Prove that any retract of a Hausdorff space is a closed set.

   (b) Let $a \in A$. Show that $f_* : \pi_1(X, a) \rightarrow \pi_1(A, a)$ is surjective.

6. Let $p : X \rightarrow Y$ be a covering map, where $X$ and $Y$ are path connected and locally path connected, and let $x_0 \in p^{-1}(y_0)$. Prove the Unique Path Lifting Theorem: Suppose $f : [0, 1] \rightarrow Y$ is any path with initial point $y_0$. Then there exists a unique lift $\tilde{f} : [0, 1] \rightarrow X$ of $f$ such that $\tilde{f}(0) = x_0$. 