1. Prove that every infinite subset of a compact Hausdorff space has a limit point.

2. Prove that if X is a connected, countable, Hausdorff, normal space then X is a one-point space.

3. Let K and L be compact subsets of topological spaces X and Y, respectively. If W is an open set in \( X \times Y \) with \( K \times L \subseteq W \), show that there are open sets \( U \) in \( X \) and \( V \) in \( Y \) with \( K \times L \subseteq U \times V \subseteq W \).

4. (a) Let \( A \) and \( B \) be subsets of a topological space \( X \) such that \( A \cup B \) and \( A \cap B \) are both connected. If \( A \) and \( B \) are both closed in \( X \), prove that \( A \) and \( B \) are both connected.

   (b) Is the hypothesis that \( A \) and \( B \) be closed really needed? Prove or give a counterexample.

5. Given subsets \( A \) and \( B \) of connected spaces \( X \) and \( Y \), respectively, with \( A \neq X \) and \( B \neq Y \), prove that \( (X \times Y) - (A \times B) \) is connected.