1. (a) Define what it means for a topological space to be compact (in terms of coverings by open sets).

(b) Prove that $X$ is compact if and only if every collection of closed sets in $X$ with the finite intersection property has a nonvoid intersection.

2. Let $X$ and $Y$ be topological spaces and assume that $X \times Y$ has the product topology.
Let $p : X \times Y \to X$ be the projection. Prove or give a counter example for each statement:

(a) $p$ is open.

(b) $p$ is closed.

(c) If $X$ and $Y$ are both connected then $X \times Y$ is connected.

3. Let $R$ be an equivalence relation on a topological space $X$ and let $p : X \to X/R$ denote the projection to the set of equivalence classes. There is the quotient topology $\mathcal{T}_q$ on $X/R$ defined by $p$. Let $\mathcal{T}$ be an arbitrary topology on $X/R$ that satisfies the following property: Given any function $g : X/R \to Y$, $g$ is continuous (with respect to $\mathcal{T}$) if and only if the composition $g \circ p$ is continuous.

Must $\mathcal{T}$ be the quotient topology $\mathcal{T}_q$? Prove or give a counter example.

4. Prove or give a counter example for each statement:

(a) A compact subspace $A$ of a space $X$ is closed in $X$.

(b) Let $X$ be a compact space and $Y$ be Hausdorff space. Every continuous map $g : X \to Y$ is also a closed map.

5. Let $\{X_\alpha | \alpha \in J\}$ be an indexed family of topological spaces. Prove that $\text{Cl}(\prod_{\alpha \in J} A_\alpha) = \prod_{\alpha \in J} \text{Cl}(A_\alpha)$ in $\prod_{\alpha \in J} X_\alpha$. 