1. Let Ω be the interior of the square with vertices at the points 1, i, −1, and −i. Show that there is a one-to-one holomorphic map $h : Ω \to \mathbb{D}$ satisfying $h(0) = 0$, and $h'(0) > 1$.

2. Show that an entire function $f$ is a nonconstant polynomial if and only if $f$ is proper, i.e., for every constant $M > 0$, the set $\{ z; |f(z)| \leq M \}$ is compact.

3. Prove or disprove the statement: Let $G = \mathbb{D} \setminus \{0, i/2\}$. Then, the holomorphic automorphism group Aut($G$) consists of the identity map only; that is, if a holomorphic map $f : G \to G$ is one-to-one and onto, then $f(z) = z$ for all $z \in G$.

4. Evaluate the integral $\int_{-\infty}^{\infty} e^{ipx} - qx^2 \, dx$, where $p, q \in \mathbb{R}$ and $q > 0$. Justify your reasoning. (You can use $\int_{-\infty}^{\infty} e^{-x^2} \, dx = \sqrt{\pi}$ without justification.)

5. Let $g$ be a holomorphic function on $\mathbb{D} \setminus \{0\}$, and denote $g_m(z) = g(z/m)$ for each positive integer $m$. Suppose that $\{g_m\}_{m=1}^{\infty}$ has a subsequence $\{g_{mk}\}_{k=1}^{\infty}$ which is uniformly bounded by 1 on the circle $\{ z; |z| = 1/2 \}$, i.e.,

$$\max_{|z|=1/2} |g_{mk}(z)| \leq 1 \quad \text{for all } k \geq 1.$$

Show that $g$ can be extended to a holomorphic function on $\mathbb{D}$.

6. Prove or disprove the statement: Let $f$ be a holomorphic on $\mathbb{D}$ and continuous on $\overline{\mathbb{D}}$. Suppose that for a constant $0 < \delta < 1/10$, $f(e^{i\theta}) = 1 + 2i$ for all $-\delta \pi < \theta < \delta \pi$. Then $f \equiv 1 + 2i$ on $\overline{\mathbb{D}}$. 