1. Characterize all those analytic functions defined in the unit disc $\Delta$ with the property that, for all $a, b \in \Delta$, $f(ab) = f(a)f(b)$.

2. Prove that there does not exist a 1-1 analytic function mapping an annulus onto a punctured disc.

3. Evaluate $\int_{|z|=1} \frac{z^{11}}{12z^{12} - 4z^9 + 2z^6 - 4z^3 + 1} \, dz$ and justify all steps. Hint: one of the ways to approach this problem is to make the change of variable $w = \frac{1}{z}$.

4. Suppose the sequence $\{f_n\}$ of 1-1 analytic functions converges uniformly on compact subsets of a region $\Omega$ to a function $f$. Show that $f$ is analytic, and is either constant or is also 1-1.

5. Let $\Omega$ be a bounded, simply connected domain in the plane. Suppose $g : \Omega \rightarrow \Omega$ is holomorphic and not the identity. Show that $g$ can have at most one fixed point.
   (a) First show it when $\Omega$ is the unit disc. Then
   (b) Show it when $\Omega$ is a bounded, simply connected region in the plane.

6. Evaluate the integral $\int_0^\infty \frac{\log x}{x^2 + a^2} \, dx$ where $a$ is real and positive.

7. Prove that if $f$ is a non-constant entire function then $f(\mathbb{C})$ is dense in $\mathbb{C}$. (You cannot just quote Picard’s Theorem.)