Exam number:________________

Math 310
Preliminary exam August 2007

1. State and prove a theorem that applies to the inital value problem

$$x' + \sin(x) = 0, x(0) = 1$$

. Explain how the theorem applies to this case.

2. Let U be the operator on the Hilbert space $L^2(0,2)$ that takes the function $f(t)$ into $t^2 f(t)$.

   Prove U is a bounded operator. Find its operator norm, (justify). Is the operator self-adjoint? Is it compact?

3. Find a Green's function for the differential operator $Ax = x'' - x$ with the boundary conditions $x'(0) = 0, x'(1) = 0$

4. (a) Find the operator norm of the Green's operator on L^2 if $Ax = x'' - x$ with boundary conditions $x'(0) = 0, x'(1) = 0$.

   (b) State and prove an existence and uniqueness theorem that applies to the equation

$$x'' - x + \lambda \cos(x) = 0, x'(0) = 1, x'(1) = 0$$

5. Prove that if an operator is of the form $A = I + K$ where K is compact, then A is injective imples A is surjective.

6. Is the map $F : L^3(0,1) \to R^1$ defined by $F(u) = \int_0^1 (u(x)^3 dx$ Frechet differentiable? (Justify your answer.) If yes, identify the derivative.