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Math 310 Preliminary Examination August , 2004

Problem 1: (20 pts)

(a) State and prove an existence and uniqueness theorem for the equations

$$\begin{aligned} x' &= f(x,y) , \\ y' &= g(x,y) , \end{aligned}$$

with initial conditions $x(0) = a$ and $y(0) = b$ under the assumptions that f, g, and all their partial derivatives are continuous.

(b) For the system:

$$\begin{aligned} x' &= x(1-x-y) , \\ y' &= y(1-2x-3y) , \end{aligned}$$

with $x(0) = y(0) = 1/10$. Can either $x(t)$ or $y(t)$ become 0 at finite time? Justify your reasoning.

Problem 2: (20 pts)

(a) Find the Green's function for the operator A where

$$Ay = -y'' + y$$

for y satisfying $y'(0) = y'(1) = 0$.

(b) find the norm of the Green's operator from L^2 to L^2..

(c) Show that there exists a unique solution for

$$\begin{aligned} -y'' + y &= \lambda \tan^{-1} y + \cos x \\ y'(0) = y'(1) &= 0 \end{aligned}$$

for an appropriate range of $|\lambda|$.

Problem 3: (20 pts)

Find a fundamental solution for the ordinary differential operator $\frac{d^2y}{dx^2} - y$. Express the solution of of $\frac{d^2y}{dx^2} - y = \phi$ as a integral.

Problem 4: (20 pts)

Define $M : L^2(0,2) \to L^2(0,2)$ such that for all $f \in L^2(0,2)$,

$$(Mf)(t) = (1 + \sin t)\ f(t)$$

Find the operator norm of M (justify fully your answer).. Show that M is not a compact operator.

Problem 5: (20 pts)

Prove that if an operator is of the form $A = I + K$ where K is compact, then A is injective implies A is surjective.

Problem 6: (20 pts)

Find the distributional derivative U_{xt} if $U(x,t) = H(x)H(t)$ with H the Heaviside step function.