310

NUMBER

~~Name:~~ ____________________

Math 310 Preliminary Examination August , 2001

Answer all 5 questions.

Problem 1: (20 pts) Show that if p, q are continuous near zero, there exists an interval $(-\epsilon, \epsilon)$ and functions u_1 and u_2 such that every solution of

$$y'' + p(x)y' + q(x)y = 0$$

can be written as a combination of u_1 and u_2 on $(-\epsilon, \epsilon)$. Can the function x^3 solve this equation?

Problem 2: (20 pts)
(a) Find the Green's function g for the operator $Ly = -y'' + y$ with the boundary conditions $y'(0) = y'(1) = 0$.
(b) Find the eigenvalues of the associated Green's operator:

$$(Gf)(x) = \int_0^1 g(x,t)f(t)\,dt$$

Problem 3: (20 pts)
Find the adjoint boundary value problem if

$$Lu = u''' + u'' + u$$

with boundary conditions $u(0) - u'(0) = 0$, $u''(1) = 0$, and $u(1) = 0$.

Problem 4: (20 pts)
Let E and F be two Hilbert spaces and $T : E \to F$ be a bounded linear operator. If there exists a complete orthonormal sequence $\{\phi_n(x),\ n = 1, ..., \infty\}$ in E such that

$$\sum_{n=1}^{\infty} \|T\phi_n\|^2 < \infty ,$$

then show that T is compact.

Problem 5: (20 pts)
Let $U(x,y)$ be equal to 1 if $x \geq 0$ and $y \geq 0$, and zero otherwise. Find $\partial^2 U/\partial x \partial y$ as a distribution.