

I'll consider two realization problems: (1) When is a torsion-free finite rank ring R the endomorphism ring $R = E(G)$ of a torsion-free finite rank abelian group G ? (2) When is a finite dimensional rational algebra A of the form $Q \otimes E(G)$ for a quotient divisible group G .

In (1), a torsion-free finite rank abelian group is just a subgroup of Q^n for some finite n and a torsion-free finite rank ring R is just a ring with underlying additive group $(R,+)$ torsion-free finite rank. The first problem was completely solved by Corner's Theorem, proved in 1967. This theorem is by now a standard tool in the study of torsion-free groups. I'll give an application from the work of Arnold and Vinsonhaler-O'Meara with regard to the separative cancellation question: Does $G \oplus G \cong G \oplus H$ imply $G \cong H$ for tffr groups?

In (2), a quotient divisible group is a mixed abelian group G with a finite rank free subgroup F such that G/F is a divisible torsion group. Vinsonhaler and Wickless give a partial answer to (2)-a large class but not all rational algebras can be so realized. Again, I'll consider the implications here with respect to separative cancellation of quotient divisible groups.