

IMPORTANT: All answers must include either supporting work or an explanation of your reasoning. These elements are considered part of the answer and will be graded.

1. (20 pts) For each part, if the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. For each T/F question, write a careful and clear justification or describe a counterexample. [5 problems]

(a) $\int (\sec(3x))^2 dx = 3 \tan(3x) + C.$ (a) T F

Give justification:

(b) $\frac{x^3}{x^2-1}$ can be written in the form $\frac{A}{x-1} + \frac{B}{x+1}.$ (b) T F

Give justification:

- (c) Assume that a force of 50 lbs is required to hold a spring stretched 2 feet beyond its natural length. The work done is stretching this spring an additional 3 feet, from 2 to 5 feet, beyond its natural length is 150 ft-lbs. (c) T F

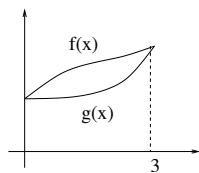
Give Justification:

- (d) Using integration by parts with $u = x$ and $dv = e^{x^3} dx$ to evaluate $\int xe^{x^3} dx$ is a reasonable choice. (d) T F

Give justification:

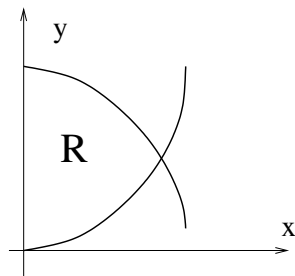
- (e) Imagine that the region between the graphs of f and g is rotated about the x -axis to form a solid. The volume of this solid is given by $\pi \int_0^3 (f(x))^2 - (g(x))^2 dx.$ (e) T F

Give justification:



2. (10 pts) Evaluate $\int (\sin(x))^2 (\cos(x))^5 dx$

3. (10 pts) Write a definite integral that gives the volume of the solid obtained by rotating the region R shown below about the y -axis. (R bounded by $y = 2 - x^2$, $y = x^2$ and $x = 0$.) (DO NOT EVALUATE THIS INTEGRAL.)



4. (10 pts) Water flows from the bottom of a storage tank at a rate of $r(t) = \frac{100e^{-\sqrt{t+1}}}{\sqrt{t+1}}$ gallons per minute. Find the amount of water that flows from the tank during the first 15 minutes. Give an exact answer, no approximations.
5. (10 pts) Find the exact area between the curve $y = x - 1$ and the curve $x = 3 - y^2$. Give an exact answer, no approximations.
6. (10 pts) Evaluate $\int \frac{1}{4-x^2} dx$ using partial fractions. (Show all steps.)
7. (10 pts) Evaluate $\int \frac{x^3}{\sqrt{4-x^2}} dx$ using trigonometric substitution. (Show all steps.)
8. (10 pts) A spherical tank of radius 8 feet is half full of oil that weights 50 lbs per cubic foot. Write a definite integral that gives the work required to pump the oil out through a hole in the top of the tank. (DO NOT EVALUATE THIS INTEGRAL.)
9. (10 pts) Write a definite integral that gives the volume of a solid whose base is bounded by the lines $y = 1 - x/2$, $y = -1 + x/2$, and $x = 0$. The cross-sections perpendicular to the x -axis are equilateral triangles. (Area of an equilateral triangle with side length L is $\frac{\sqrt{3}}{4}L^2$. (DO NOT EVALUATE THIS INTEGRAL.)
10. (10 pts) Points will be given for the clarity of your writing and the mathematical correctness of your explanations.
- (a) Explain why $\int_0^4 \frac{1}{\sqrt{4-x}} dx$ is an improper integral.
- (b) Determine whether or not the integral in (a) converges. If it converges, then compute its limit; otherwise, explain why it diverges.
11. (10 pts)
- (a) Find $\int \cos(x)e^{-\sin(x)} dx$
- (b) Determine whether or not the integral $\int_0^\infty \cos(x)e^{-\sin(x)} dx$ converges. If it converges, then compute its limit; otherwise, explain why it diverges.