Final Exam A **Math 116** Spring 2008

Name:______________________________ Section:__________

Pg 1 ____ Pg 2 ____ Pg 3 ____ Pg 4 ____ Pg 5 ____ Pg 6 ____ Total ______

IMPORTANT: All answers must include either supporting work or an explanation of your reasoning. These elements are considered part of the answer and will be graded.

1. (15 pts) For each part, if the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. For each T/F question, write a careful and clear justification or describe a counterexample. [5 problems]

(a) $\displaystyle\sum_{n=2}^{\infty}(\frac{4}{5})^n = 5.$ (a) T F

Justification:

(b) If $b_n > 0$ and $b_{n+1} < b_n$ then the series $\displaystyle\sum_{n=1}^{\infty}(-1)^n b_n$ converges. (b) T F

Justification:

(c) $y = x^2 + x$ is a solution to $y' = 2(y - x^2) + 1$ (c) T F

Justification:

(d) If $\displaystyle\sum_{1}^{\infty} a_k x^k$ converges at x=1 and x=2 then it converges at x = -2. (d) T F

Justification:

(e) $\dfrac{2-i}{1+i} = \dfrac{1}{2} - \dfrac{3}{2}i$ (e) T F

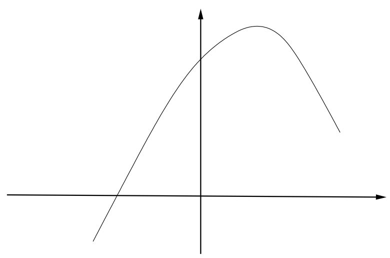
Justification:

2. (5 pts) The function $f(x)$, whose graph is shown, has the Taylor polynomial of degree 2 about $x = 0$ given by $P_2(x) = a + bx + cx^2$. What can you say about a, b, c? Circle the answers.

(i) a is ☐ negative, ☐ zero or ☐ positive

(ii) b is ☐ negative, ☐ zero or ☐ positive

(iii) c is ☐ negative, ☐ zero or ☐ positive



3. (5 pts) Find the slope of the line tangent to the parametric curve $x = t\cos t$, $y = 3t + t^5$ when $t = 0$.

4. (5 pts) Convert the polar coordinates $(5, \frac{3\pi}{4})$ into Cartesian coordinates.

5. (10 pts) (a) Find a power series that represents the function $\dfrac{1}{1-x}$ and give its radius of convergence?

(b) Use part (a) to find a power series that represents the function $\dfrac{x}{1+x^3}$ and give its radius of convergence?

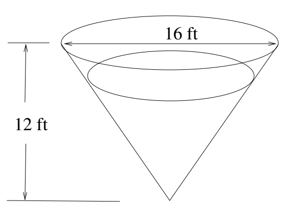
6. (10 pts) A tank initially contains 100 gallons of brine in which 50 lb of salt are dissolved. A brine containing 2 lb/gal of salt runs into the tank at the rate of 5 gal/min. The mixture is kept uniform by stirring and flows out of the tank at the same rate of 5 gal/min.

   (a) If $y(t)$ is the amount of salt in the tank after t minutes, write down the initial value problem describing the mixing process:

   $y' =$ $y(0) =$

   (b) Find $y(t)$, the amount of salt in the tank after t minutes.

7. (10 pts) A conical tank (shown below) has a height of 12 feet and the diameter of the top is 16 feet. It is filled to within 2 feet of the top with olive oil weighing 57 lb/ft^3. How much work does it take to pump the oil to the rim of the tank? Give your answer as a definite integral. DO NOT EVALUATE.

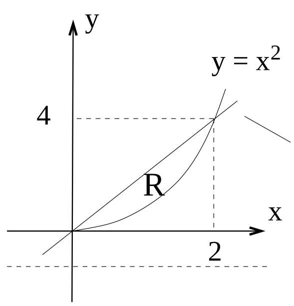


8. (10 pts) Use $\displaystyle\int \frac{1}{(x+1)^{4/3}}\ dx = \frac{-3}{(x+1)^{1/3}} + C$ to find the value of the following improper integrals, or, if an integral does not converge, say so explicitly and show this.

(a) $\displaystyle\int_1^{\infty} \frac{1}{(x+1)^{4/3}}\ dx$

(b) $\displaystyle\int_{-1}^{1} \frac{1}{(x+1)^{4/3}}\ dx$

9. (10 pts) Consider the region R bounded by $y = 2x$ and $y = x^2$. A solid figure P has R as a base region and cross-sections perpendicular to the x-axis are squares. Express the volume of P as a definite integral. DO NOT EVALUATE.



10. (10 pts) (a) Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{2^n}{n}(x-3)^n$.

(b) Determine at which endpoints of the interval of convergence the series converges and write down the interval of convergence. Justify your conclusions.

11. (10 pts) Write a definite integral that represents the area inside the region enclosed by one loop of the graph of $r = 2\sin(3\theta)$. **DO NOT EVALUATE**

12. (10 pts) Determine whether the following series converges or diverges. Circle the correct answer and then justify your answer.

$$\sum_{n=1}^{\infty} \frac{n^2}{3^n}$$ CONVERGES DIVERGES

Justification:

13. (10 pts) Determine whether the following series converges absolutely, converges conditionally or diverges. Circle the correct answer and then justify your answer.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{n^4+2}}$$ CONVERGES ABSOLUTELY CONVERGES CONDITIONALLY DIVERGES

Justification:

14. (10 pts) Use the integral test to determine whether the following series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{ln(n)}{n^2}$$