

Name: _____

Section: _____

Page 1 ____ Page 2 ____ Page 3 ____ Page 4 ____ Page 5 ____ Total _____

IMPORTANT: All answers must include either supporting work or an explanation of your reasoning. These elements are considered part of the answer and will be graded.

1. (15 pts) For each part, if the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. For each T/F question, write a careful and clear justification or describe a counterexample. [5 problems]

(a) $\int_1^{\infty} x^{-3/2} dx = 1.$

(a) T F

Give justification:

(b) The function $f(x) = \frac{\ln(x)}{x}$ is a solution of the differential equation $x^2y' + xy = 1.$

(b) T F

Give justification:

(c) The length of the curve $x = \cos t, y = t + \sin t, 0 \leq t \leq \pi$ is equal to $\int_0^{\pi} \sqrt{2 + 2 \cos(t)} dt.$

(c) T F

Give Justification:

(d) If the parametric curve $x = f(t), y = g(t)$ satisfies $f'(1) = 4$ and $g'(1) = 0$, then it has a horizontal tangent when $t = 1.$

(d) T F

Give Justification:

(e) The graph of $x = 2 \cos(t), y = \sin(t)$ is an ellipse.

(e) T F

Give Justification:

2. (10 pts) Points will be given for the clarity of your writing and the mathematical correctness of your explanations.

(a) Explain why $\int_0^4 \frac{1}{\sqrt{4-x}} dx$ is an improper integral.

(b) Determine whether or not the integral in (a) converges. If it converges, then compute its limit; otherwise, explain why it diverges.

3. (10 pts)

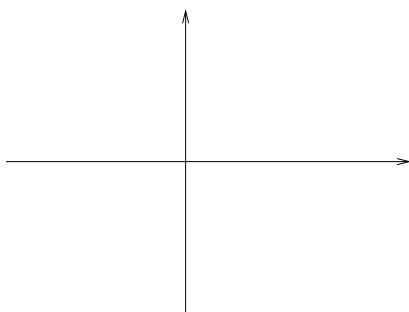
(a) Find $\int \frac{1}{x(\ln(x))^2} dx$

(b) Determine whether or not the integral $\int_2^\infty \frac{1}{x(\ln(x))^2} dx$ converges. If it converges, then compute its limit; otherwise, explain why it diverges.

4. (10 pts) Find the area of the surface formed by revolving the graph of $f(x) = x^3$ on the interval $[0, 1]$ about the x -axis.

5. (10 pts) Consider $r = 1 + \sqrt{2} \cos(\theta)$.
 (a) Sketch the graph of $r = 1 + \sqrt{2} \cos(\theta)$ and give the coordinates of the five points at which the curve crosses the coordinate axes. (Hint: There is an inner loop and an outer loop.)

List Coordinates



r	θ

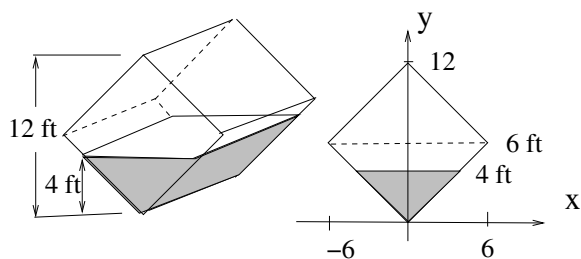
- (b) Write a definite integral that represents the area inside the region enclosed by the inner loop. **DO NOT EVALUATE**

6. (10 pts) Use Euler's method with step size 0.5 to estimate $y(1.5)$, where y is the solution to the initial-value problem $y' = \frac{y}{x}$, $y(0) = 2$. (Hint: Calculate the first three approximations in Euler's method.)

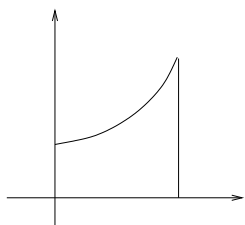
7. (5 pts) A metal oil tank has cross-section that is a square rotated 45° as shown in the figure. Its height is 12 ft and width is 12 feet. The oil in the tank has a weight-density of 57 lb/ft^3 .

(a) Write a definite integral that expresses the fluid force on the end of the tank when the oil is 4 feet deep? **DO NOT EVALUATE**

(b) (Bonus 5 pts) Now assume the tank is full. Express the fluid force on the end of the tank as a sum of two definite integrals. **DO NOT EVALUATE**



8. (5 pts) Suppose (\bar{x}, \bar{y}) is the center of mass (or the centroid) of the region bounded by the curves $y = e^x$, $y = 0$, $x = 0$ and $x = 1$. Express \bar{x} and \bar{y} using definite integrals. **DO NOT EVALUATE**



$$\bar{x} =$$

$$\bar{y} =$$

9. (5 pts) A 200 gal tank is filled with fresh water (no salt in the water). Brine containing 1/2 lb of salt per gallon of water runs into the tank at a rate of 5 gal/min. The well-stirred mixture is pumped out of the tank the same rate of 5 gal/min. Write a differential equation with an initial condition that is satisfied by the function $y(t)$ giving the amount of salt in the tank after t minutes? **DO NOT SOLVE**

Answer:

$$\frac{dy}{dt} = \qquad y(0) =$$

10. (10 pts) Use separation of variables to solve the initial value problem $\frac{dy}{dt} = 3t^2e^{-y}$, $y(0) = 1$.

11. (10 pts) A national park is known to be capable of supporting 100 grizzly bears, but no more. Ten bears are in the park at present. We model the population with a logistic differential equation $\frac{dP}{dt} = kP(1 - \frac{P}{K})$ with $k = 0.001$.

(a) Find a formula for $P(t)$.

(b) When will the bear population reach 50?