

Name: _____

Section: _____

Page 1 ____ Page 2 ____ Page 3 ____ Page 4 ____ Page 5 ____ Total _____

IMPORTANT: All answers must include either supporting work or an explanation of your reasoning. These elements are considered part of the answer and will be graded.

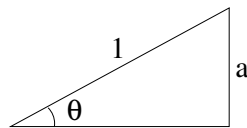
1. (20 pts) For each part, if the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. For each T/F question, write a careful and clear justification or describe a counterexample. [5 problems]

(a) $\frac{d}{dx}(|x^2 - 4|) = |2x|$. (a) T F
Give justification:

(b) The function $f(x) = x^7 - 8x^6 + 5$ has a zero in the interval $(0, 1)$. (b) T F
Give justification:

(c) If $\lim_{x \rightarrow 2} [f(x)/g(x)]$ exists then it must be $f(2)/g(2)$. (c) T F
Give Justification:

(d) If θ is the angle illustrated in the figure (where the lengths of two sides of a right triangle are shown) then $\theta = \arctan \frac{a}{\sqrt{1-a^2}}$ (d) T F
Give Justification:



(e) $\lim_{x \rightarrow 0} \frac{x}{\sin(6x)} = 6$. (e) T F
Give Justification:

2. (7 pts) The height h (in feet) of a ball t seconds after being thrown up into the air from a hot air balloon is given by $h(t) = -16t^2 + 64t + 192$.

(a) Find the velocity of the ball at time t and give units.

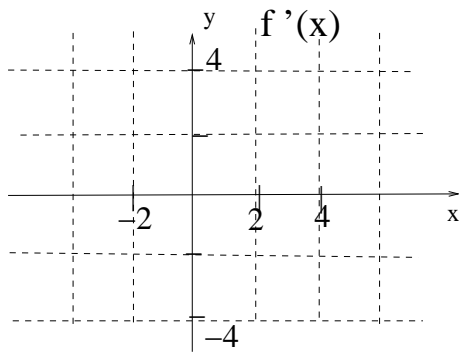
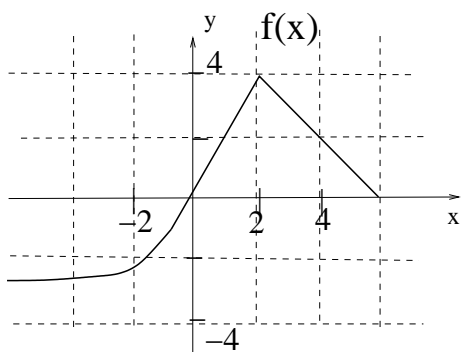
(b) What is the height of the ball when it stops ascending and starts descending?

3. (8 pts) Use the properties of limits to calculate $\lim_{x \rightarrow -3} \left(\frac{x+3}{x^2+4x+3} \right)^2$. Show all steps.

4. (8 pts) Use the chain rule to find the derivative of the function $y = 4(\sin(2x))^3$.

5. (5 pts) If $f(\pi/8) = 0$ and $f'(x) = 3/2 + (\cos(2x))^2$, find the equation of the tangent line to the graph of this function at the point $(\pi/8, 0)$.

6. (7 pts) Sketch the graph of $f'(x)$ for the function $f(x)$ given:



7. (15 pts) (a) Write the limit definition for the derivative $f'(x) =$

(b) Use the limit definition of the derivative and algebra to find the derivative of the function $f(x) = \sqrt{4x + 1}$.

8. (5 pts) Find the horizontal asymptote(s) for the function $f(x) = \frac{8x^4 + 5x^3}{16 + 7x^3 - x^4}$ by computing the appropriate limit(s). (Show work.)

9. (15 pts) $f(x) = \begin{cases} 5x - x^3 + k & \text{for } x \geq 2 \\ \frac{5x^2 - 10x}{x-2} & \text{for } x < 2 \end{cases}$

(a) Find $\lim_{x \rightarrow 2^-} f(x)$.

(b) Find $\lim_{x \rightarrow 2^+} f(x)$.

(c) There is a constant k that makes the function $f(x)$ above continuous at $x = 2$. Find the number k .

(d) Use limits and the **definition of continuity** to explain why this choice of k makes $f(x)$ continuous everywhere.

(e) With this choice of k , will $f(x)$ be differentiable at $x = 2$? Explain your reasoning.

10. (10 pts) Given $g(2) = 3$ and $g'(2) = -4$:

(a) If $f(x) = \frac{x^2 + g(x)}{g(x)}$ find $f'(2) = \underline{\hspace{2cm}}$

(b) If $f(x) = e^{g(x)}$ find $f'(2) = \underline{\hspace{2cm}}$