Practice Final Exam **Math 116** Spring 2008

Name:_____ Section:_____

IMPORTANT: All answers must include either supporting work or an explanation of your reasoning. These elements are considered part of the answer and will be graded.

1. For each part, if the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. In each case, write a careful and clear justification or a counterexample.

(a) If the power series $\sum C_n(x+1)^n$ diverges at $x=1$ and converges at $x=-3$ then the radius of converge must be equal to 2.. (a) T F

(b) The Taylor series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n}x^n$ converges to e^{-x} for all x. (b) T F

(c) The sequence $a_n = (1+\frac{1}{n})^{2n}$ converges to e. (c) T F

(d) If $f(x)$ is a bounded continuous function with the properties that $f(x) > \frac{1}{x}$ for $1 \le x < 50$, $f(50) = \frac{1}{50}$, and $0 < f(x) < \frac{1}{x^2}$ for $x > 100$, then $\int_1^{\infty} f(x)\,ds$ diverges. (d) T F

(e) $\sum_{n=1}^{\infty}(\frac{5}{6})^n = 5$.

(e) T F

2. Suppose $f(3) = 4,\ f'(3) = 5,\ f''(3) = -2,\ f'''(3) = 12$ and all derivatives of f are continuous. Write down the third degree Taylor polynomial $P_3(x)$ for $f(x)$ about $x = 3$.

3. Convert the polar coordinates $(6, 7\pi/3)$ into Cartesian coordinates.

4. Set up an integral that represents the length of the curve $x = t^4 - 3$, $y = 3\sin t + t^2$ when $2 \le t \le 3$.

5. Use separation of variables to solve the initial value problem $\dfrac{dy}{dx} = e^{y-x}$ with $4y(\ln 2) = -\ln 2$.

6. The walls of a storage tank are obtained by rotating the curve $y = 2x^{3/2}$ (for $0 \le x \le 4$) around the y-axis. (Units are in feet.) The tank is filled to the top with water.
(a) Write an integral (do not evaluate) giving the volume of water contained in the tank when it is full.
(b) Water is pumped out over the top of the tank at the rate of $8 - .25t$ ft^3/min. after pumpinging for t minutes. Write an integral (do not evaluate) that gives the work done in pumping all the water from the tank.

7. Find the interval of convergence of the power series $\sum_{n=1}^{\infty}\dfrac{n^3}{2^n}(x-3)^n$.

8. Construct the Taylor Polynomial of degree 3 for $f(x) = \arctan x$ about $x = 0$.

9. Find the value of the following improper integrals, or, if an integral does not converge, say so explicitly and show this.

(a) $\int_0^3 \frac{1}{(x-1)^{2/3}}\ dx$ (b) $\int_1^\infty \frac{\ln(x)x}{x^2}\ dx$

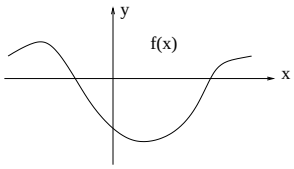
10. A diligent student has a slow leak in her bike tire, but has been too busy studying for exams to fix it. Assume that the pressure in the tire decreases at a rate proportional to the difference between the atmospheric pressure (15 lbs) and the tire pressure. On Monday at 6:00 PM she pumped the tire to a pressure of 85 lbs. By 6:00 PM Tuesday it was down to 75 lbs. Let $P(t)$ denote the tire pressure t days from Monday at 6:00 PM.
(a) Write down the initial value problem whose solution is $P(t)$.
(b) Solve the differential equation and use the given initial conditions to find an expression for $P(t)$.

11. (a) Sketch the graph of $r = 1 + \sqrt{2}\cos\theta$ and give the coordinates of the points at which the curve crosses the coordinate axes.
(b) Write a definite integral (do not evaluate) that represents the area of the region inside the inner loop of the limacon $r = 1 + \sqrt{2}\cos\theta$.

12. (a) Write down the Taylor series for $\frac{1}{1+x}$ about $x = 0$.
(b) Use your result from part (a) to write down the Taylor series about $x = 0$ for $\ln(x+1)$.

13. A solid figure P has a base region $\{(x,y)|x^2+y^2 \le 4\}$ in the xy-plane. Every cross-section of P by a plane perpendicular to the x-axis is an equilateral triangle. Find the volume of P.

14. The graph of a function $f(x)$ is shown below (graph will show $f(0) < 0, f'(0) < 0, f''(0) > 0$). Which of the following could be the Taylor polynomial approximating $f(x)$ for x near 0? More than one answer is possible. Justify your answer.



(a) $P_2(x) = 2 + 2x + 2x^2$ (b) $P_2(x) = 2 - 2x + 2x^2$ (c) $P_2(x) = 2 + 2x - 2x^2$
(d) $P_2(x) = 2 - 2x - 2x^2$ (e) $P_2(x) = -2 + 2x + 2x^2$ (f) $P_2(x) = -2 - 2x + 2x^2$
(h) $P_2(x) = -2 - 2x + 5x^2$ (i) $P_2(x) = -2 - 2x - 2x^2$ (j)$P_2(x) = -2 + 2x - 2x^2$

15. Determine whether each of the following series converges absolutely, converges conditionally or diverges and then briefly explain WHY for each series.

(a) $\sum_{n=1}^{\infty}(-1)^n n e^{-n^2}$ (b) $\sum_{n=2}^{\infty}(-1)^n \frac{\sqrt{n}}{2n^3-7}$ (c) $\sum_{n=1}^{\infty}(-1)^n(1+\frac{1}{n^2})$