Name:_____ Section:_____

IMPORTANT: All answers must include either supporting work or an explanation of your reasoning. These elements are considered part of the answer and will be graded.

1. For each part, if the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F.

   (a) A critical point is always a local minimum or maximum. (a) T F

   (b) A continuous function defined on all points of a closed interval always has a global maximum and a global minimum. (b) T F

   (c) A critical point of f' could occur at an inflection point of f. (c) T F

   (d) If $f'' = 0$ at $x = 0$, then the graph of f changes concavity at $x = 0$. (d) T F

   (e) If $f' > 0$ on an interval, the function is concave up on the interval. (e) T F

   (f) If f is always decreasing and concave down, then f must have at least one root. (f) T F

2. Consider the function $f(x) = x \ln x$, $x > 0$. List the critical points of f. [Give the x values. If there are none, write none.]

3. On what interval is $f(x) = \ln(x^2 + 1)$ concave up?

4. The foot of the ladder shown in the figure moves away from the wall at a speed of 2 ft/min, causing the top of the ladder to slide down the wall without leaving it.
   (a) $\frac{dx}{dt}$ and $\frac{dy}{dt}$ have the same sign.
   (b) The top of the ladder is moving faster and faster.
   (c) Keeping $\frac{dx}{dt}$ constant, doubling x and y and the length of the ladder, doubles $\frac{dy}{dt}$.

5. The volume of a certain tree is given by $V = \frac{1}{12\pi}C^2h$, where C is the circumference of the tree at the ground level and h is the height of the gree. If C is 5 feet and growing at the rate of 0.2 feet per year, and if h is 22 feet and is growing at 4 feet per year, find the rate of growth of the volume V.

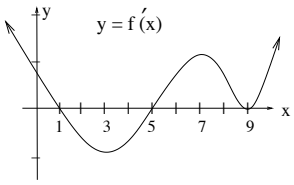
6. Find the equation of the tangent line to the curve given by $f(x) = x \sin x$ at the point $x = \pi/2$.

7. Let $f(x)$ be a function with positive values and let $g(x) = \sqrt{f(x)}$.
   (a) If f is increasing at $x = x_0$, what about g?
   (b) If f is concave down at $x = x_0$, what about g?
   (c) If f has a local maximum at $x = x_0$, what about g?

8. A landscape architect plans to enclose a 3000 square-foot rectangular region in a botanical garden. She will use shrubs costing \$25 per foot along three sides and fencing cosing \$20 per foot along the fourth side. Find the dimensions that minimize the cost.

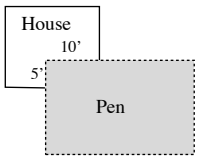
9. A rectangular sheet of paper is to contain 72 square inches of printed matter with 2 inch margins at top and bottom and 1 inch margins on the two sides. What dimensions for the sheet will use the least paper?

10. The graph of the **derivative** $y = f'(x)$ of $f(x)$ is shown below. (**Caution:** This is not the graph of $f(x)$ itself)
(a) On what intervals is $f(x)$ concave down?
(b) For which x-values does $f(x)$ have local maximum values?



11. Let g be a function such that $g(2) = 4$ and whose derivative is known to be $g'(x) = \sqrt{x^2 + 1}$.
(a) Use a linear approximation to estimate the value of $g(1.95)$. Show your work.
(b) Do you think your estimate in part (a) is an overestimate or an underestimate? Explain.

12. A rectangular chicken pen is to be fenced in with one side centered along the 25 foot long wall of a chicken house (as shown in the figure). There is 280 feet of fencing available and no fencing is required along the portion of the pen next to the chicken house. What dimensions should the pen be to enclose the largest possible area?



13. Given $g(3) = 9$ and $g'(3) = -3$:
(a) If $f(x) = \dfrac{x^3}{g(x)}$ find $f'(3) =$
(b) If $f(x) = (g(x))^{1/2}$ find $f'(3) =$
(c) If $f(x) = (x^2 + 2)e^{g(x)}$ find $f'(3) =$

14. The number of bacteria in milk grows exponentially once the milk has been bottled. When the milk is put into the bottles, it has an average bacteria count of 600 million per bottle. Eight days later the bacteria count has grown to 900 million per bottle.
(a) Write an equation for $f(t)$, the number of bacteria t days after the milk is bottled.
(b) At what rate is the bacteria count growing after 12 days? (Include units with your answer.)

15. The table gives values for functions f and g, and their derivatives. Determine the given derivatives and write down the formulas used.

x	0	1	2
$f(x)$	5	-1	4
$g(x)$	2	2	0
$f'(x)$	-2	2	3
$g'(x)$	5	6	0.5

(a) If $K(x) = \dfrac{f(x)}{g(x)}$ then $K'(0) =$ _____?
(b)If $H(x) = f(g(x))$ then $H'(1) =$ _____?
(c)If $F(x) = e^{f(x)}$ then $F'(1) =$ _____?
(d)If $G(x) = (f(x))^2$ then $G'(1) =$ _____?
(e)If $K(x) = \sin(g(x))$ then $K'(2) =$ _____?