

IMPORTANT: All answers must include either supporting work or an explanation of your reasoning. These elements are considered part of the answer and will be graded.

1. For each part, if the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. In each case, write a careful and clear justification or a counterexample.

(a) If $f(x)$ is continuous for all x then $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ exists for all x . (a) T F

Justification:

b) If $f(x) = 3^x$ then $f^{-1}(x) = \frac{\ln(x)}{\ln(3)}$ (a) T F

Justification:

(c) $\arcsin\left(\frac{1}{3}\right)$ is an angle whose sine is $\frac{1}{3}$. (a) T F

Justification:

(d) The function $y = 3 + 6e^{-kt}$, with k a positive constant, has a horizontal asymptote of $y = 6$. (b) T F

Justification:

(e) If $f(x) = \frac{5}{e^x}$ then the derivative $f'(x)$ is decreasing for all x . (a) T F

Justification:

f) If a function is not continuous at a point, then it is not defined at that point. (a) T F

Justification:

2. (a) Evaluate **using properties of limits and algebra:** $\lim_{x \rightarrow -3} \frac{2x^2 - 18}{x + 3}$

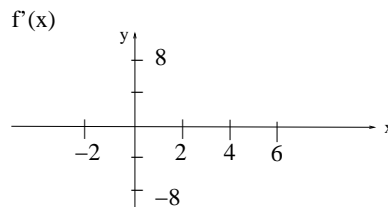
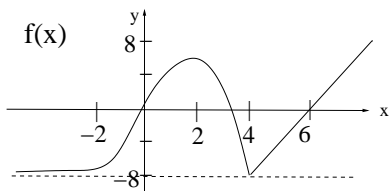
(b) Is it possible to choose k so that the following function is continuous? Explain your reasoning.

$$f(x) = \begin{cases} \frac{2x^2 - 18}{x + 3}, & \text{if } x \neq -3; \\ k, & \text{if } x = -3. \end{cases}$$

(c) Sketch the graph of $f(x)$ in (b) with $k = 2$.

(d) Can one choose k so that the above function is differentiable at $x = -3$? Explain.

3. Given the graph of the function below:



(a) Sketch the graph of its derivative function.

(b) What is the value of $f''(5)$?

(c) What do we know about $f'(4)$?

4. In 1990, the population of Mexico was about 84 million and approximated by $P(t) = 84(1.026)^t$ (in millions), where t is in years since 1990.
- (a) What was the rate of growth in the year 1995 (give the units)?
- (b) In what year will the population reach 150 million?

5. Use the table of values for f, f', g and g' to find:

(a) If $K(x) = \frac{f(x)}{g(x)}$ then $K'(4) = \underline{\hspace{2cm}}$?

(b) If $H(x) = f(x)g(x)$ then $H'(-2) = \underline{\hspace{2cm}}$

x	f(x)	f'(x)	g(x)	g'(x)
-2	3	5	7	2
3	0	-4	1	3
4	8	6	-1	5

6. (a) Write the limit definition for the derivative of $f(x) = \frac{1}{x} + 3$.
- (b) Find $f'(x)$ **using algebra and the limit definition** in (a). No credit will be given for simply using the formulas for writing down the derivative.
7. A toy rocket is launched upward from the roof of a building 112 feet high. It rises and then falls back; its height above the ground t seconds after it is thrown is $y = -16t^2 + 96t + 112$ feet, until it hits the ground.
- (a) Find the average velocity of the rocket during the first 2 seconds? Give units.
- (b) When does the rocket hit the ground and how fast is it going at that time? Give units.
8. Given $f(x) = \frac{(x+1)(1+3x^2)}{(x+2)(x^2+3x+2)}$.
- (a) Use limit rules to find $\lim_{x \rightarrow -1} f(x)$
- (b) Determine the horizontal and vertical asymptotes of the graph of $f(x)$.
9. Describe how the Intermediate Value Theorem can be used to show that there must be a root of the equation $y = x^3 + x - 5$ in the interval $(1, 2)$.
10. Let $f(x) = e^{x^3-3x+1}$.
- (a) Find the equation of the tangent line at the point $(0, e)$.
- (b) Find all points of the graph of $f(x)$ where the tangent line is horizontal.